ON

THE THEORY OF THE MOON,

AND ON

THE PERTURBATIONS OF THE PLANETS.

BY

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Sed neque quàm multæ species, nec nomina quæ sint,
Est numerus; neque enim numero comprendere refert:
Quem qui scire velit, Libyci velit æquoris idem
Discere quàm multæ zephyro turbentur arenæ:
Aut, ubi navigiis violentior incidit eurus,
Nôsse, quot Ionii veniant ad litora fluctus.

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PREFACE.

M. Plana published, in 1832, his great work on the Lunar Theory, which was printed at the expense of the King of Sardinia. M. Plana made use of the method given by Laplace in vol. iii. of the Méc. Cél., which consists in employing the equations in which the true longitude is the independent variable, and the expressions for the variables, r, nt, and s, being first obtained in terms of the true longitude, by the reversion of series, r, v, and s are afterwards found in terms of the mean longitude, nt. The distinguishing and important feature of M. Plana's work is, that the expressions for the coefficients are maintained to the last in a literal form: this condition gives facilities for their verification which cannot, in my opinion, be too highly appreciated.

Shortly after the publication of M. Plana's work, I began to consider this subject; and between 1834 and 1840 I published the first four parts of this work. I endeavoured to obtain the coefficients directly by means of the equations of motion, which had always been employed in determining the perturbations of the planets, and in which the time is the independent variable; still maintaining the literal development, as in M. Plana's work.

About the same time, or shortly afterwards, my friend M. de Pontécoulant began to follow in the same track, and to employ the methods which I had introduced. His work was completed and published in 1846; and I look upon it as one of the most valuable ever produced in the history of astronomy, whether from the importance of the results which it contains, or from the very great labour and perseverance which were required to

obtain them. M. de Pontécoulant carried the approximations further than I had done, and as far as M. Plana; so that, although in some instances it may be desirable to carry the approximations even further, it will be seen presently, that unquestionably when this work appeared, it furnished the means, as far as the coefficients of the periodic terms are concerned, of constructing tables, and of obtaining from those tables places of the moon more accurate than those of Burckhardt or any which had preceded them, within the limits of the errors of observations made at sea, and even within those of the observations made at Greenwich. The work of M. de Pontécoulant contained, therefore, the solution of the problem of finding the moon's place from theory alone, sufficiently exact to be used for finding the longitude at sea, a solution which all the greatest mathematicians have been aspiring after, ever since the time of Newton.

In 1848, the Astronomer Royal published the "Reduction of the Greenwich Lunar Observations," and with it the accurate determination of the elements of the moon's orbit; a work which has greatly contributed to the perfection of the lunar tables: for all that theory can do is, from certain given quantities or constants to deduce the rest; theory can throw no light on the longitude of the epoch, the eccentricity, etc.

I have been reproached with not having made lunar tables; but the expense of their construction and publication is very considerable, and those who make lunar tables "are in kings' courts." The publication of my Lunar Theory has cost me a considerable sum of money already; if I had published lunar tables, I must have incurred an expense of at least £1,000 more; and if I had made tables, I think it probable they would have shared the fate of Damoiseau's tables, and never have been used. The American tables, if not identical with such tables as the coefficients published by M. de Pontécoulant would furnish, are so close to them, that we can form as clear a notion of the merits of those coefficients, as if we possessed tables solely and immediately constructed from them.

In 1853, Prof. Peirce, in America, appears to have considered it desirable, from the data furnished by the Astronomer Royal and the coefficients in Plana's work, to construct tables to be used for finding the places of the moon given in the American *Nautical Almanac*. The title of this work is as follows: "Tables of the Moon; constructed from Plana's Theory, with Airy's and Longstreth's Corrections, Hansen's Two Inequalities of Long Period arising from the Action of Venus, and Hansen's Values of the Secular Variations of the Mean Motion and of the Motion of the Perigee. Arranged in a Form designed by Professor Benjamin Peirce, Consulting Astronomer, for the Use of the Nautical Almanac, under the Superintendence of Charles Henry Davis, Lieut. United States Navy, and published under the Authority of the Hon. John P. Kennedy, Secretary of the Navy. Washington: printed for the Use of the Nautical Almanac. 1853."

Of the 94 coefficients employed in forming the expression for the moon's longitude in the American Tables, 74 are said to be taken from Plana; but those are the coefficients upon which Plana and Pontécoulant agree, and therefore they belong as much to one as the other. The coefficients of Plana which Pontécoulant had ascertained to be erroneous, were avoided by Prof. Peirce through the fortunate interposition of Mr. Longstreth, of Philadelphia, at the last moment, as will presently be explained; and in eight out of eleven instances the values of Pontécoulant were employed instead, which had been published six years previously by Pontécoulant, having been obtained empirically by Mr. Longstreth.

Eight coefficients are due to the Astronomer Royal, having been obtained by him empirically, and published in vol. xvii. of the Memoirs of the Astronomical Society, viz.,

The coefficient of the	equation of the centre	$= 22639''.2$.
,,	variation	$= 2371.0$.
,,	evection	$= 4586.9$.
,,	annual equation	$= 670.3$.

The coefficient of the equation of the centre is an arbitrary constant, which must be determined from observation. M. de Pontécoulant employed Burckhardt's value, which is $22639''{\cdot}7$. M. de Pontécoulant's values of the coefficients of variation and evection, obtained from theory, are identical with the values of the Astronomer Royal, obtained empirically: these also, therefore, belong properly to M. de Pontécoulant.

The next inequalities taken from the Astronomer Royal are the very small inequalities,

$$1''{\cdot}0 \sin(nt-\xi-\eta)+''{\cdot}7 \sin(nt+\xi-\eta)$$

The inequalities due to the ellipticity of the earth are taken from the Astronomer Royal:

$$6''{\cdot}38 \sin(nt-\eta)-{\cdot}97 \cos(nt-\eta).$$

The coefficient of the first, according to Plana, is $6''{\cdot}6$; according to Pontécoulant, vol. iv. p. 486, $6''{\cdot}623$.

The Astronomer Royal says: "The latter term is not recognised by theory; but I conceive that there cannot be the smallest doubt of its reality, and that its value differs very little from the above" ("Memoirs of the Astronomical Society," vol. xviii. p. 38). The former term might also have been taken from M. de Pontécoulant with equal advantage.

We now come to Mr. Longstreth's empirical corrections; they are eleven in number (see p. 25): of these, eight are identical with those which M. de Pontécoulant had published six years previously, and correspond to coefficients of Plana which he had marked as erroneous. It is impossible, therefore, to avoid having a suspicion that Mr. Longstreth's attention was called to these coefficients by M. de Pontécoulant's remarks, although no mention is made of M. de Pontécoulant's work; and it is curious that the Americans, instead of adopting M. Plana's coefficient (Arg. 22), which is $3''{\cdot}309$, and marked as erroneous by M. de Pontécoulant, adopt $''{\cdot}8$, attributing it to Plana, which is close to the value, $''{\cdot}9$, given by M. de Pontécoulant. The coefficient of Plana, Arg. 67, which is one of those which Longstreth deter-

mined, although not marked by Pontécoulant, differs considerably from his value, which may account for Longstreth's examination of it. His determination is identical with Pontécoulant's value.

The following is the only information I have been able to obtain respecting Mr. Longstreth's coefficients, and it is contained in the Appendix to the American Nautical Almanac for 1855:

"After the ephemeris for the present year was almost wholly completed, the following empirical corrections of Plana's formula for the Moon's longitude were received from Mr. Miers Fisher Longstreth, of Philadelphia; and notwithstanding the consequent labour and delay, the corresponding changes of the ephemeris were computed and applied.

"*Longstreth's Corrections of the Moon's Longitude.*

$=3''\!\cdot\!6 \sin(2\tau-2\xi+\xi_{,})$	$*-1''\!\cdot\!4 \sin(2\tau+\xi_{,})$
$*+1\cdot4 \sin(2\tau-2\xi-\xi_{,})$	$*+1\cdot1 \sin(\xi+\xi_{,})$
$*+2\cdot2 \sin(\xi-2\eta)$	$+1\cdot0 \sin(4\tau+\xi)$
$*-3\cdot4 \sin(4\tau-2\xi)$	$+1\cdot8 \sin(4\tau-2\xi-\xi')$
$*-2\cdot4 \sin(2\tau+2\eta)$	$*+0\cdot5 \sin(2\tau-\xi-2\eta)$."
$*-2\cdot8 \sin(2\tau-\xi-\xi_{,})$	

It is evident that Prof. Peirce attached considerable importance to Mr. Longstreth's work, or he would not have incurred the great labour and delay of recomputing the Moon's places.

In those cases which I have marked with an asterisk, Mr. Longstreth adopted the values of M. de Pontécoulant.

The following are the four cases in which M. de Pontécoulant's coefficients differed the most from those employed in the American Tables:

	Pont., vol. iv. p. 624.	Damoiseau's Tables.
Arg. 5. —	$''668\cdot9 \sin \xi_{,}$	$''673\cdot0$
27. —	$1\cdot3 \sin(2\tau-2\xi+\xi_{,})$	$\cdot7$
133.	$1\cdot3 \sin(4\tau+\xi)$	$1\cdot9$
	$1\cdot2 \sin(4\tau-2\xi-\xi_{,})$	$3\cdot0$

M. de Pontécoulant has recently, by pushing the approximation further, corrected his previous determinations, and obtained the following values:

Arg.	27.	$1''{\cdot}5 \sin(2\tau-2\xi+\xi_{\prime})$	$2''{\cdot}2$ Longstreth.
	133.	$1{\cdot}7 \sin(4\tau+\xi)$	$1{\cdot}9$ Longstreth.
		$2{\cdot}1 \sin(4\tau-2\xi-\xi_{\prime})$	$3{\cdot}0$ Longstreth.

In two of these three cases, Mr. Longstreth has taken Damoiseau's values, probably not knowing that in the case of Arg. 27, Damoiseau corrected his former coefficient and reduced it in his Tables to ·7. The reason why the coefficient of the annual equation differs, is, that the Americans took the value determined by the Astronomer Royal empirically in 1848, viz., —670″·3; but the Astronomer Royal's recent determination is 669″·00: the value now assigned to it by M. de Pontécoulant is —″668·3.

The following are the differences which exist at present between M. de Pontécoulant's coefficients in the expression for the longitude and those employed in the American Tables arranged in their order of magnitude:

	P.—A.	The Americans have taken the coefficients from
Arg. 5.	$2''{\cdot}0 \sin \xi_{\prime}$	Airy, 1848.
	$-{\cdot}9 \sin(4\tau-2\zeta-\xi_{\prime})$	Longstreth.
Arg. 27.	$-{\cdot}7 \sin(2\tau-2\xi+\xi_{\prime})$	Longstreth.
Arg. 9.	$-{\cdot}6 \sin(2\tau-2\xi)$	Plana.
Arg. 2.	${\cdot}5 \sin \xi$	Airy, 1848.
Arg. 6.	$-{\cdot}5 \sin(2\tau-\xi_{\prime})$	Plana.
Arg. 14.	$-{\cdot}4 \sin(\xi-\xi_{\prime})$	Plana.
Arg. 15.	${\cdot}3 \sin(2\tau-\xi+\xi_{\prime})$	Plana.
Arg. 39.	$-{\cdot}3 \sin(2\tau-4\xi)$	Plana.
Arg. 101.	$-{\cdot}3 \sin \tau$	Plana.
Arg. 132.	${\cdot}3 \sin(4\tau-\xi)$	Plana.

Neglecting all, of which the coefficient is less than ″·3.

The above, therefore, is the quantity which should be added to the longitude of the American tables, in order to obtain the longitude that would be given by the coefficients of M. de Pontécoulant, when quantities under $''\cdot 3$ are neglected.

These differences are so small, that practically the places given by the American tables may be considered as identical with those which would be given by tables founded upon Pontécoulant's coefficients, and therefore as founded upon our theory, with the exception of Hansen's two erroneous equations due to the action of Venus. The period of these last is so long, that they may be considered as merely affecting the longitude of the epoch; and the Astronomer Royal's three empirical corrections not embraced by theory are so small, as not sensibly to affect the result.

The Americans profess to have employed the Astronomer Royal's value of the inclination given in the "Reduction of the Greenwich Observations," which is $18535''\cdot 46$; the Astronomer Royal's recent determination of this quantity, viz., $18535''\cdot 55$, is so close to the former, that the difference may be neglected. The Americans profess, in every other instance, to have adopted Plana's coefficients: but this expression for the latitude, like that given by the Astronomer Royal in the introduction to the Greenwich Lunar Reductions, appears to be derived from Plana's expression in terms of the true longitude; hence, the errors pointed out in p. 55, and which arose in the process of converting that expression into terms of the mean longitude do not exist in the American Tables.

Captain Davis has kindly informed me that, although Mr. Longstreth's equations were introduced into the ephemeris for 1855, Prof. Peirce's tables were first used for the ephemeris of 1856.

If we make the Moon's mass

$\frac{1}{75}$	the Astronomer Royal's value of the Parallactic inequality	$8''\cdot8103$
$\frac{1}{67\cdot3}$	gives for the Solar Parallax	$8''\cdot8406$

values which appear large.

The two coefficients of Prof. Hansen rest solely upon the author's unsupported assertion; which, considering how difficult they are to obtain, is very unsatisfactory. Indeed, Prof. Hansen admits that the accurate deteimination of these two inequalities by theory, is the most difficult matter which presents itself in the theory of the moon's motion. As it is now thirteen years since Prof. Hansen published the values of these inequalities, there has been ample time for him to furnish some details respecting the manner in which he obtained them. According to Prof. Hansen, it will always be preferable to deduce the values of the mean motions of the perigee and the node from the observations. "There will, therefore, be found in his tables, with reference to these two quantities, an empiricism, similarly as in the case of the two inequalities of long period" mentioned above. I infer from this sentence, and from the paper in the Ast. Nach., that Prof. Hansen obtained these coefficients empirically; but, however difficult it may be to obtain them from theory, I apprehend, considering how imperfect the observations were during the last century and previously, it is far more difficult, if not impossible, to obtain them from the observations empirically.

Prof. Hansen states, that he slightly altered these coefficients (see the *Monthly Notices* for Nov., 1854). The Americans took them from No. 597 of the *Ast. Nachrichten*, where they are given thus:

$$27''\cdot4 \sin(-g-16g'+18g''+35^\circ\ 20'\ 2'')$$
$$23\ \cdot2 \sin(8g''-13g'+315^\circ\ 30').$$

g	being the mean anomaly of the		Moon,
g'	,,	,,	Earth,
g''	,,	,,	Venus.

But in the Lunar Tables recently published by Prof. Hansen, they are given as follows:

$$15''\cdot34 \sin(-g-16g'+18g''+30^\circ\ 12')$$
$$21''\cdot47 \sin(8g''-13g'+274^\circ\ 14').$$

That is to say, Prof. Hansen has entirely abandoned the values which the Americans adopted from his Memoir in the *Ast. Nach.*

The differential equations which govern the Lunar Theory have been long known, but the mechanical difficulties of their application are so great, that the greatest mathematicians have been arrested by them; and although the great practical importance of the subject has been appreciated ever since the time of Newton, it has been reserved for me to make the announcement in this treatise, that we at last possess the means, through the labours of M. Plana, M. de Pontécoulant, and myself, of forming Tables of the Moon from theory alone, and without having recourse to any empirical assumptions, sufficiently correct for the purposes of navigation, and as correct as any which we at present possess. The empirical tables, however, at present in use, are so accurate, that there is very little room for improvement. The errors of the observations of the moon at Greenwich, vary up to $\pm''6$, or more when they are made under unfavourable circumstances, as in twilight, and the small differences which exist between the place given by the American Tables and the observations at Greenwich (A—O) are due, at least as much to the errors of the observations as to the error of the tabular places. This is confirmed by the extreme irregularity of the differences. And it should be remarked that the large differences which occasionally occurred before, have been entirely got rid of by the American Tables no less than by those of Prof. Hansen. If we except the Parallactic inequality, of which the value obtained empirically by the Astronomer Royal differs more than 2″ from our value, there is now, I believe, no coefficient in the longitude of which the value is doubtful to the amount of 1″. The difficulties which we have had

to encounter, and which we have at last successfully surmounted, arise from the innumerable number of terms of the higher orders; and although each, taken separately, is so small as hardly to produce a sensible influence upon the result, to obtain the requisite degree of accuracy, an enormous number must be taken into account. This leads to an amount of computation which can only be fully appreciated by those who have gone through it; indeed, the difficulties of the literal solution are so great that they have been pronounced by Prof. Hansen to be insurmountable.

Unfortunately for us, the manner in which the Americans have described the tables in the Introduction, deprives my friend M. de Pontécoulant and myself of all the credit to which we are entitled; all our coefficients which they employ, they attribute to Plana or to Airy when they have an excuse for doing so; and when they have not, Mr. Longstreth appropriates them to himself, affixing the initial of his name to them.

With respect to the coefficients of the American Tables taken from Prof. Hansen, I believe the tables would be better without them. The inequalities due to Venus have been abandoned by their author; and they would affect the accuracy of the tables, were it not that they are mixed up with the longitude of the epoch determined by the Astronomer Royal when he believed them to be correct, and the error of the one is compensated by the error of the other.

Poisson states, in his Memoir of 1833 on the Theory of the Moon, that he had ascertained that the coefficient of the inequality, of which the period is 240 years, did not, if it existed at all, exceed $''\cdot 3$.

The following are the words of Poisson:

"M. Airy ayant trouvé récemment que l'action de Vénus sur la terre produit, dans le mouvement apparent du soleil, une inégalité sensible dont la periode est de 240 ans, cette inégalité, en tant qu'elle affecte l'excentricité de l'orbite solaire, doit se trouver aussi dans l'équation séculaire de la lune, et produire, dans sa

longitude moyenne, une inégalité dont la période est également de 240 ans. Il restait donc à savoir si le coefficient de cette inégalité a une grandeur sensible. Pour m'en assurer, j'ai eu recours à l'obligeance de M. G. de Pontécoulant, qui a calculé, de son côté, l'inégalité découverte par M. Airy: je l'ai prié de me communiquer la partie de cette inégalité, relative à l'excentricité de l'orbite solaire; et j'ai reconnu que l'inégalité correspondante dans le mouvement de la lune ne s'élève qu'à un ou deux centièmes de seconde; ce qui la rend tout à fait negligeable."

And again, in p. 98:

"M. de Pontécoulant ayant calculé, de son côté, cette inegalité solaire, et ayant été conduit exactement au résultat que M. Airy avait annoncé je l'ai prié de me communiquer la partie de son calcul relative à l'inégalité de e'; en la substituant dans la formule précédente, il en est résulté

$$\delta\epsilon = (0''\cdot026026)\, m \sin\lambda - (0''\cdot28072)\, m \cos\lambda\,;$$

λ étant treize fois la longitude moyenne de Venus, moins huit fois celle de la terre. Or, m étant un peu moindre qu'un treizième, cette inégalité ne s'élève pas à un quarantième de seconde dans son maximum, et peut en conséquence, être négligée."

M. de Pontécoulant verified this result; and it has recently been confirmed by M. Delaunay.

The coefficient of the other inequality, of which the period is 273 years, has been determined from theory by M. Delaunay, in the Conn. des Temps for 1862, and the value he assigns to it is

$$16''\cdot336 \sin(-g - 16g' + 18g'' + 35^\circ\ 16'\cdot5)$$

Since writing the above,* I have met with a short paper by Mr. Longstreth, in the Transactions of the American Philosophical Society of Philadelphia, for 1853, entitled, *On the Accuracy of the Tabular Longitude of the Moon, to be obtained by the Construction of New Lunar Tables. By Miers Fisher Longstreth.*

* I did not meet with this paper of Mr. Longstreth, until after the first 32 pages had been printed off.

The following extract gives all the information which the paper contains:

"The discovery of the inequalities of a long period, by Hansen, together with the Reduction of the Greenwich Lunar Observations, from 1750 to 1830, afford ample materials for the construction of new lunar tables, and lead to the inquiry, what additional accuracy can be obtained? The coefficients deduced from theory by Damoiseau, Plana, Pontécoulant, and those deduced from observation by Burckhardt (though differing considerably), give the moon's place with nearly the same accuracy. Where a difference exists, I have carefully compared them with observation, and deduced the most probable value. To test the accuracy of the new coefficients thus obtained, I have selected from the Reduction of Greenwich Lunar Observations all the observations made during the years 1820, '21, '23, '24, and '25, numbering 499, and have computed the moon's place with the new coefficients, by correcting Plana's when necessary; they having been used in the Reduction of Greenwich Lunar Observations to obtain the moon's tabular place. In the following pages, I have arranged for comparison the errors of Plana's coefficients and those of the new coefficients, to which have been added the corrections for Hansen's inequalities, and most of the corrections required by Plana's theory deduced by G. B. Airy. Upon examination, it will be found that in many cases where the errors of the new coefficients are large, that the observations have been made while the sun was above the horizon, or during twilight. The errors here given are necessarily compounded of the errors of the tabular place, and of those of observation."

Then follow some remarks upon the probable limits of error of the Greenwich observations; and he shows, by reference to the observations, that the error of observation often amounts to from 3″ to 6″. The author then gives a table of the errors of Plana's coefficients, as compared with the errors of 499 Right Ascensions of the Moon, calculated by means of what he calls the new coefficients; but he gives no details of the method he

adopted in forming these coefficients; and there is nothing to contradict the supposition that he merely selected from Damoiseau, Plana, and Pontécoulant the coefficient which appeared to him likely to produce the best result. I have supposed Mr. Longstreth calculated Right Ascensions, but this is left in doubt: one column of the table is headed, "Errors of Plana's Coefficients," and the other, "Errors of New Coefficients."

A comparison of the places given by the American Tables is published in the *Astronomical Journal*, No. 129, made by Mr. Newcomb, with those of the Greenwich observations for the years 1856 and 1857, from which it appears that the mean error in R. A. for 1856 is 3″·52; for 1857, 2″·76: the mean error in declination is 2″·03. Prof. Hansen states the mean error of R. A., from his tables, to be 2″·44. Professor Hansen multiplies his coefficients in the expression for the longitude by arbitrary factors, and diminishes his latitudes arbitrarily by 1″, upon the ground that the Moon's centre of gravity does not coincide with the centre of figure, a supposition purely gratuitous, and the reasoning which is founded upon it is incorrect. Therefore, however accurate they may be, I can only consider Prof. Hansen's tables as a retrograde step in science. But the above are not the only empiricisms with which Prof. Hansen's tables are affected. Professor Hansen tells us, that the mean motions of the perigee and node employed in his tables are empirical; that is, they "have been deduced from the observations"; and "there will, therefore, be found in the tables, with reference to these two quantities an empiricism, similarly as in the case of the two inequalities of long period." See Prof. Hansen's Letter to the Astronomer Royal, in the *Monthly Notices* for November, 1854.

M. Plana has justly observed that the Tables of Prof. Hansen must take their place with those of which Euler said:—"Non tam *Theoria* quam observationibus sunt superstructæ": "Huiusmodi ergo Tabularum sive consensus, sive dissensus cum observationibus, neque ad Theoriam Newtonianam plenissime confirmandam, neque ad eam infringendam allegari potest; nam quatenus

istae Tabulae observationibus satisfaciunt, hoc non solum Theoriae est tribuendum; quatenus autem cum observationibus minus conveniunt, hoc ne Theoriae quidem imputari potest; propterea quod istae Tabulae non soli Theoriae innituntur."

An empirical solution, however, is not susceptible of the same degree of accuracy as my method, which admits of unlimited verification in detail. The facility of tracing any error is due to the system of exhibiting all successive steps of calculation in such a form that their connexion can be readily examined. But, above all, it is to be regretted, that by using auxiliary variables, and not finally giving expressions for the longitude and latitude of the moon, Prof. Hansen's results shed no light upon anything which has been done before, and nothing that has been done before sheds any light upon his results; so that his work is almost a solitary instance in the annals of science to which the following sentence does not apply,

Λαμπάδια ἔχοντες διαδώσουσιν ἀλλήλοις.

There is no error in M. Plana's analytical expression for the longitude in terms of the first, second, third, or fourth order. There are three errors of the fifth order, of which one is insensible.

In M. Plana's analytical expression for the reciprocal of the radius vector, there is no error in terms of the first, second, or third order, and only one in terms of the fourth order.

In M. Plana's analytical expression for the latitude, there are no errors of the first, second, or third order, and only two of the fourth order.

By carefully examining Mr. Plana's numerical reductions, M. de Pontécoulant and myself have succeeded in discovering the causes of the principal differences between M. Plana's coefficients and those of M. de Pontécoulant; but there remain many discrepancies in terms of the fifth and the higher orders, which it is very desirable should be got rid of. M. de Pontécoulant and myself, where we differ from M. Plana, went

over our calculations so many times, that we think the error can hardly be on our side; and, unless M. Plana will revise his work, we must despair of this ever being accomplished; for no one again will ever possess the intimate knowledge which M. Plana has of the indirect method.

As it appeared to me that astronomers would view with greater confidence a comparison of places given by the American Tables, made by persons who could have no interest in enhancing their value, I made application to Mr. Hind, the Superintendent of the *Nautical Almanac*; and, in consequence, he directed Mr. Farley to procure places of the moon from the American Almanac, and compare them with the observations made at Greenwich for the years 1856, 1857, and 1858; and as Mr. Hind has kindly allowed me to publish them with this treatise, any one can see at once how extremely accurate the places given by these tables are, and how much more so than places given by Burckhardt's tables. These tables, which appear to be as accurate as Prof. Hansen's, were published in 1853; and, therefore, they were in existence when the Astronomer Royal stated to the Astronomical Society, in reference to Prof. Hansen's tables published in 1857, that "probably in no recorded instance has practical science ever advanced so far in accuracy by a single stride." They were in existence, also, when the Board of Visitors passed their Resolution asserting that Prof. Hansen's Tables "had been found, by an extensive comparison with the Greenwich Observations, far to surpass in accuracy any previous ones." The falsehood and injustice of this Resolution are such, that the Visitors must have been ignorant of the existence of the American Tables. These are so close to Prof. Hansen's, that it would be unfair to one or the other to give the palm of superiority to either in point of accuracy, unless after a careful comparison with the *same* Observations. No trifling superiority, however, in the places given by Prof. Hansen's empirical tables, if it should be found to exist, ought to be allowed to compensate for the differ-

ence in the foundation upon which they rest. How close they are, is shown by the following specimen:

Greenwich Mean Time.			Moon's Right Ascension.					Moon's Declination.					
			Burckhardt.			Han.	Am.	Burckhardt.				Han.	Am.
1860.	d.	h.	h.	m.	s.	s.	s.		°	′	″	″	″
July	18	0	7	46	59·38	57·07	57·62	N.	21	52	17·6	21·3	20·3
		1	7	49	29·73	27·40	27·94		21	42	29·8	33·8	32·8
		2	7	51	59·84	57·50	58·03		21	32	33·1	37·3	36·4
		3	7	54	29·72	27·36	27·89		21	22	27·6	32·1	31·2
		4	7	56	59·36	56·98	57·51	N.	21	22	13·4	18·1	17·2

These American Tables, founded on theory, are invaluable, now that the Astronomical Society and the Board of Visitors have attached so much importance to empirical tables, and that we are threatened to be carried back to the time of Clairaut. For in these Tables, coefficients are employed, with only three exceptions, and those of little moment, founded upon our labours; that is, M. Plana's, M. de Pontécoulant's, and my own, and due to theory alone. I am confident, therefore, that a just posterity will give, not to the Americans who employed our coefficients, nor to Prof. Hansen who published tables in 1857, but to us, that is, to Plana, Pontécoulant, and Lubbock, who in 1846 furnished the means of constructing tables of the moon without any empirical hypothesis, the credit of first bringing the errors of the lunar theory within the limits of the errors of observation, and thereby of bringing to perfection the solution of the problem of finding the longitude at sea by means of lunar observations.

Partly at the request of my friend M. de Pontécoulant, and partly because I thought it desirable to show how such terms are to be obtained when the equations are employed which I have used in this work (that is, in which the time is the independent variable), I have calculated the much controverted term $\frac{3771}{64} m^4 e_{\prime}^2$ in the secular equation of the moon's mean motion, following step by step the method given in p. 133, Part II. The

calculation may appear longer than by the other method used by Mr. Adams; but I have given more details and more intermediate steps: Mr. Adams' paper is very concise.

At page 4, I have made a remark, in the words of Mr. Adams; upon the circumstance of my friend M. de Pontécoulant arriving at two different values of the coefficient of $m^4e_{\prime}^2$ in the secular inequality of the moon's mean motion; and having communicated this treatise to him as it came from the printer, I received from him the following remarks:

" J'ai, sans doute, mal expliqué ma pensée ou j'aurai eu le malheur d'être mal compris par vous, car vous donnez à mes paroles une interprétation très diffèrente de celle qu'on en doit tirer. Dans les Nos. des *Monthly Notices*, ou des *Comptes-Rendus* de l'Académie des Sciences, où j'ai traité ce sujet, j'ai dit: 'Qu'ayant repris en entier le calcul de *l'équation séculaire* du moyen mouvement lunaire d'après les formules de Laplace, où la longitude vraie de la lune est prise pour la variable indépendante, afin de vérifier l'expression donnée par M. Plana dans sa théorie de la lune, j'étais parvenu à un résultat parfaitement concordant avec le sien, et qui n'en diffère légèrement que dans les termes d'un ordre très élevé qui ne peuvent avoir aucune influence sensible sur les coefficients numériques. J'ai ajouté qu'ayant comparé ce résultat avec celui que j'avais obtenu par un calcul direct, c'est-à-dire, en faisant usage des formules où le temps était pris pour la variable indépendante, j'aí reconnu, non sans *surprise*, comme vous le dites, mais en me rendant à l'évidence, que ce dernier résultat était absolument différent de celui qu'on obtient en faisant usage de la première méthode, et j'en ai conclu qu'il était indispensable dans la recherche du coefficient de *l'équation séculaire*, de recourir aux formules directes et de renoncer tout-à fait à l'emploi des formules usitées par tous les géomètres qui jusqu'à vous et moi s'étaient occupés de la théorie de notre satellite, comme ne pouvant conduire qu'à des résultats fautifs et erronés. Quant à l'idée d'avoir égard à la variation de l'excentricité de l'orbite terrestre dans les formules différentielles du mouvement

troublé, émise d'abord par Poisson dans son mémoire de 1833, et reproduite depuis par M. Adams dans son mémoire de 1852, je persiste à penser qu'elle doit être repoussèe comme ne pouvant conduire qu'à des résultats tout-à-fait opposés à l'observation, mais surtout sous le rapport analytique, comme contraire au beau théoreme de *l'invariabilité des grands axes* et *des moyens mouvements* planétaires, qui, selon moi, est général, et s'étend à la lune aussi bien qu'à tous les autres corps du systéme solaire.' Cette question du reste a été amplement traitée dans un supplément à ma théorie de la lune, qui s'imprime en ce moment, et qui paraîtra d'ici à quelques jours."

When Mr. Adams and Mr. Leverrier found that Bouvard's Tables of Uranus were in discordance with the path of that planet, they set to work to discover some principle which might account for it; and so this discordance of the Tables led to the discovery of Neptune. Let us hope that the discordance which now exists between Mr. Adams's secular equation and the ancient eclipses will lead to a discovery of equal importance.

M. Plana has since, in a letter to me, dated June 19th, 1860, and published, admitted the accuracy of the coefficient of Mr. Adams.

In the following pages I have set forth the nature of the results which have been accomplished by M. Plana, M. de Pontécoulant, and myself, as well as the construction of the American Tables in which they are embodied, and which were ignored by the Astronomer Royal in his communication to the Astronomical Society (*Monthly Notices*, 8th April, 1859). We must console ourselves with the English reflection: "that, though plainness and truth are ofttimes abused with subtlety and falsehood, yet in the end alway truth triumphs, when falsehood shall take reproach."

High Elms, Farnborough, Kent,
1*st January*, 1861.

ON THE

THEORY OF THE MOON.

MR. ADAMS, in a paper on the Secular Variation of the Moon's mean Motion, printed in the Philosophical Transactions for the year 1853, gave as the coefficient of a certain term in this inequality, a numerical quantity differing from that of Plana. Since that time, Plana has revised his original calculation, and produced two other coefficients successively. Professor Hansen has arrived at a coefficient not differing much from Plana's. M. Delaunay has arrived at a result identical with Mr. Adams. M. de Pontécoulant has arrived at two results, one agreeing with Plana, and the other not; but neither agreeing with Mr. Adams. M. le Verrier takes for granted Adams and Delaunay are in error, and will not even examine M. Delaunay's calculations, because his value of the coefficient will not satisfy the ancient eclipses. So that Adams and Delaunay, agreeing with each other, maintain one value; Plana, Hansen, and Pontécoulant, disagreeing with each other, express in the most positive manner their conviction that Adams and Delaunay are wrong. I have not as yet alluded to Laplace, because, in fact, he never attempted to calculate the quantity about which this controversy has arisen.

Will it be believed, in future ages, that three of the best, if not the very best, astronomers in Europe are ranged on one side, and two on the other; the question to be decided being, whether a certain numerical coefficient is $\frac{2187}{128}$ according to M. Plana, or $\frac{3771}{64}$ according to Mr. Adams? or whether $\frac{de'}{dv}$ is identical with $\frac{de'}{ndt}$? and whether Mr. Adams, "*professeur distingué de l'Université d'Oxford,*" the discoverer

of Neptune, is ignorant of the first principles of the differential calculus, and the 8th page of the elementary treatise of Lacroix?

M. de Pontécoulant says: "Et que nous importe à nous que ses formules (that is, M. Delaunay's) soient approuvées à Londres, à Berlin ou à Saint Petersbourg? Nous avons prouvé qu'elles sont fautives, cela doit nous suffire; et pour dire ici notre pensée tout entière, nous déclarons bien haut que quand bien même les Académies de Canton, de Pékin ou de Tchoug-Kouë, se léveraient en masse pour nous assurer du contraire, elles ne changeraient pas notre opinion à cet égard."

Again: "Je ferai voir où était la faute de la formule de M. Adams, faute inconcevable, il faut le dire, comme on le verra bientôt, car elle ne porte pas, comme on pourrait le croire, sur l'une de ces abstractions si difficiles à aborder par l'esprit humain qu'elles divisent les meilleurs esprits, et qu'après d'interminables discussions, ils se séparent moins d'accord qu'auparavant. La faute de M. Adams, inaperçue et reproduite par M. Delaunay dans ses calculs, roulait, en vérité, je n'ose le dire, sur l'*un des premiers principes* du calcul differentiel, et l'on en trouve la correction dans la 8[e] page du Traité ELEMENTAIRE de Lacroix." *

The vivid imagination of my friend, M. de Pontécoulant, leads him to suppose that this controversy will extend itself even to the Celestial Empire. The following extract will show that it has already reached the classic towers of Gotha.

> Hinc movet Euphrates, illinc Germania bellum;
> Vicinæ ruptis inter se legibus urbes
> Arma ferunt; sævit toto Mars impius orbe.

Prof. Hansen says, in a letter to the Astronomer Royal, dated Gotha, May 31, 1859:—

"Delaunay's Säcularänderung der mittleren Mondlänge muss ich entschieden für unrichtig halten. Ich habe folgende drei Resultate durch die Theorie erhalten:—

(1) + 11· 93, Ast. Nach. No. 443,
(2) + 11· 47, „ No. 597,
(3) + 12·120, in den Mondtafeln angewandt.

"Worin der Fehler von Delaunay liegt, kann ich in diesem Augenblick nicht sagen, aber man kann als nicht unwahrscheinlich annehmen,

* See Dernières Observations, etc., p. 14, and p. 3.

dass bei der Entwickelung der Mondstörungen nach den Potenzen von m, Glieder, die mit *sehr hohen* Potenzen multiplicirt sind, merklich werden Können. Ueberhaupt hat man ja gar keinen Beweis von der Convergenz dieser Réihen, und sie müssen nothwendig bei vergrössertem Werthe von m diverginen. Ich habe bekanntlich diese Art der Entwickelung gar nicht angewandt, sondern von Annäherung zu Annäherung die erhaltenen Störungen in die Gleichungen substituirt. Dieses Verfahren habe ich so lange fortgesetzt, bis das Resultat der letzten Annäherung bis auf kleine Bruchtheile von Secunden dem Resultat der vorletzten Annäherung gleich wurde. Ich habe dazu nur 12 oder 13 Annäherungen nöthig gehabt."

The words of Le Verrier are these: "Nous conservons donc, nous devons le dire, puisque l'on nous y oblige, des doutes, et plus que des doutes sur les formules de M. Delaunay. Très certainement la vérité est du côté de M. Hansen. Et n'étant point disposé à suivre M. Delaunay dans la discussion indéfinie et sans doute obscure qu'il annonce, nous déclarons à l'avance que nous tenons pour nulle et non avenue toute réponse dans laquelle M. Delaunay n'établira pas que sa théorie n'est pas contredite par les observations."

M. Le Verrier is the last philosopher from whom we should have expected such a remark, as his own discovery of Neptune would have never been made, had he acted upon this theory.

In his theory of the moon, M. Plana obtained one value of the secular acceleration. In 1856, he printed a paper, in which he admitted that his theory was wrong on this point, and actually deduced Mr. Adams's result from his own equations. Soon afterwards, however, M. Plana retracted his admission of the correctness of Mr. Adams's result, and obtained a third result, differing from his former one and from that of Mr. Adams.

In p. 46 of M. Plana's Memoir upon the secular equation of the mean motion of the moon, read on the 1st June, 1856, before the Academy of Turin, there occurs this passage: "Vers le commencement du mois d'avril dernier, je n'avais pas fait la distinction entre les fonctions séculaires $\int'\zeta dv$ et $\int''\zeta dv$: mais des reflexions ulterieures m'ont persuadé qu'il fallait rejeter la fonction séculaire $\int''\zeta dv$, comme entièrement étrangère aux développemens par lesquels on forme l'équation séculaire du moyen mouvement de la lune." Until M. Plana favours us with the nature of these reflexions, it is impossible to form an opinion upon their value; and the suppression of the term $\int''\zeta dv$ wears at present an arbitrary character.

M. de Pontécoulant now gives two different values of the secular

acceleration, one of which he has obtained by using the time, and the other by using the moon's longitude as the independent variable; and he does not appear startled at obtaining two contradictory values, but seems inclined to defend both.

This state of things is surely unexampled in the history of science, more especially as it seems to be conceded by all parties, that the point where the divergence takes place is where Mr. Adams, instead of confounding the mean motion with the true longitude, as Plana does, or making

$$\frac{de'}{d\nu}=\frac{de'}{ndt}, \quad \nu \text{ being the true longitude}$$

makes

$$\frac{de'}{d\nu}=\frac{ndt}{d\nu}\frac{de'}{ndt}.$$

It is not always easy to get to the original source of error, or the passage where it became first introduced; but in the case of the secular inequality of the Moon, the first trace of it appears to me to be in Vol. III. of the Méc. Cél., p. 213, where Laplace has the equation—

$$e'=E'+fv+lv^2,$$

E' being the eccentricity of the earth's orbit at a given epoch, and v the true longitude of the Moon.

But this expression for e' would not have led to fatal consequences, if the terms depending upon m^4 and higher powers of m could safely be neglected; and if the following sentence (Méc. Cél. iii. p. 227), were admissible:—"Les termes dependans du carré de la force perturbatrice, changent un peu cette valeur de l'Equation Séculaire de la longitude moyenne; mais il est aisé de voir que les termes de cette ordre, qui ont une influence très-sensible sur l'Equation Séculaire du Perigée, n'en ont qu'une très-petite et insensible sur celle du moyen mouvement." In fact, whether Plana's coefficient be right or not, it is quite certain that the terms which Laplace supposes to have no sensible influence, entirely change the numerical value of the secular inequality; and that all Laplace's inferences, based upon the first approximation to its value, or the coefficient of m^2, fall to the ground.

Let us now consider the bearing of the question, as regards the ancient eclipses, and let us admit that the geographical co-ordinates of the central line are accurately known for the eclipses of Agathocles, Thales and Xerxes, which have formed the subject of the interesting

researches of the Astronomer Royal, in the Phil. Trans. for the year 1853.

Let us also admit, with Laplace, that analysis shows, that neither the resistance of ether nor the successive transmission of gravity produces any sensible alteration in the mean motion of the nodes or perigee of the Moon; and let us also admit, that if the value of the co-efficient of the secular inequality be that assigned to it by Laplace, the conditions of the eclipses above mentioned would be satisfied. It would follow then, according to Laplace, that the length of the day has not varied the hundredth part of a second since the time of Hipparchus. See Méc. Cél., vol. iii. p. 176, and vol. v. p. 361.

If, indeed, the value of the secular inequality given by Laplace were correct, it might serve as the foundation of other theories, as, for instance, that of the invariability of the velocity of rotation of the earth about its axis; but unless this invariability can be proved independently, it cannot be used, or the eclipses which hang upon it, to upset the value of the secular inequality which Adams has obtained.

Laplace must have been aware, that the invariability of the velocity of the rotation of the earth, was only a contingent truth, and that it was liable to be modified by forces in existence. In Vol. V. of the Méc. Cél. he gives a chapter upon the diminution of the length of the day, in consequence of the cooling of the earth. With certain values of constants, Laplace arrives at the conclusion that the length of the day has only varied in 2,000 years $\frac{1''}{237}$, and with other values $\frac{1''}{387}$. But it must be recollected that Laplace was impressed with the belief, that his value of the secular inequality of the moon precluded the possibility of any alteration of the length of the day having taken place. It would be interesting to ascertain whether, by assigning other values to the constants within reasonable limits, a different result could be obtained. Laplace has also enunciated in the most positive manner (Méc. Cel., vol. ii. p. 92), the invariability of the geographical situation of the poles; but this truth depends also upon the invariability of the form of the earth, and the immutability of the strata of which it is composed. Since the Méc. Cél. was written, the writings of geologists, and especially of Lyall and Darwin, have made us familiar with the numerous changes to which the crust of the earth has been subjected; and circumstances have transpired, such as the existence of coal and the remains of plants, as well as of animals, whose existing allies are only found in temperate or warm climates, very far to the north; facts which were not known at the time when Laplace wrote the Méc. Cél.

Professor Huxley informs me, that the most northerly large fossil reptile that has yet turned up, is the Ichthyosaurus discovered by Belcher at Exmouth Island, in lat. 77° 16′ North, and 96° West longitude. The remains are described by Owen in the Appendix to the last of the Arctic voyages. M'Clintock found a patch of lias with characteristic Ammonites, etc., and perhaps ichthyosaurian bones, at Port Wilkie, in about 76° North, and 117° West longitude. An abundant fossil Fauna, of Silurian and Carboniferous age, has been found in the same latitude, together with thick seams of coal proceeding from carboniferous plants. At Disco Island, in New Greenland, there is plenty of coal of tertiary, probably miocene age. It is impossible, if the known laws of vegetable physiology are allowed to have a retrospective application, to admit the existence of an abundant and rich Flora, with an annual darkness of six months' duration.

The memoir of Laplace, in the Conn. des Temps for 1824, is not free from numerical mistakes, which will not surprise any one conversant with the subject; for Laplace, though so great a mathematician, may not have been a skilful computer. The experience I acquired in calculating the easier terms, which are fortunately those which must be encountered first, enabled me by degrees to attempt the more difficult ones; and when I made the calculation of the coefficient of $m^3 \cos (2ct-2gt)$ (or $m^3 \cos (2\xi-2\eta)$, Arg. 77 in my notation), in the development of R given by M. de Pontécoulant in the Conn. des Temps, 1840, p. 38, to which I shall refer hereafter, I had been incessantly occupied in such operations for three years.

About the accuracy of the correction introduced by my friend Mr. Adams, I am surprised there can be any doubt; and it is very curious, that M. de Pontécoulant, who now calls this step a "supercherie analytique," detected the same mistake of M. Plana in other passages of M. Plana's great work and also in that of Damoiseau. He also detected that the value of some inequalities of long period, calculated by Laplace, in the Conn. des Temps for 1823 and 1824, were vitiated by the same error. M. de Pontécoulant's memoir on this subject is given in the Conn. des Temps for 1840.

There are several passages in this memoir of M. de Pontécoulant in which this error is correctly described; and especially the following, p. 67.

"M. Plana confond à tort, les formules où l'on prend, comme il le fait, pour variable indépendante la longitude vraie de la Lune, avec celle où la différentielle du temps est supposée constante.

"La comparaison .... prouve combien il est important de distinguer

avec soin ces deux cas, et dans quelles erreurs on peut tomber en employant, comme le fait Laplace, indifféremment les deux espèces de formules, sans bien préciser les propriétés particulières qui les distinguent. Il est sans doute étonnant qu'aucun des géomètres qui ont étudié les travaux de Laplace sur la théorie lunaire, n'ait aperçu cette grave erreur; et c'est par cette raison que nous nous sommes cru obligé, dans ce mémoire, de revenir si souvent sur le même sujet." Can anything be clearer than this, and might not any one suppose it had been written with a view to point out the error of Plana, which Adams discovered?

Again, with respect to what Adams calls the "areolar velocity":

"Laplace dit que dans cette formule, on peut supposer r^2dv proportionnel à l'élément du temps même, en ayant égard aux termes de l'ordre m^3; on verra plus loin que cela n'est point exact." P. 30.

Again: "Cette différence tient à ce que dans la première approximation les deux équations, $\delta R = o$ et $\int d' \delta R = o$, que suppose l'analyse de Laplace ont lieu en effet, soit qu'on suppose la fonction δR exprimée en fonction de v ou en fonction de t, mais dans la seconde approximation elles ne subsistent ni l'une ni l'autre, du moins en général, par rapport à la variable v; et la seconde seule à lieu dans tous les cas, par rapport à la variable t. Laplace, en employant indifferemment les formules où la longitude vraie est prise pour variable indépendante et celles où l'on suppose constant l'element du tems, sans faire cette importante distinction, ne pourrait arriver qu'à des résultats erronés toutes les fois qu'il sortait de la première approximation. Nous en verrons encore d'autres exemples dans la suite." P. 42.

I received many letters from M. de Pontécoulant about the time this memoir was published, in which this question is much dwelt upon; the following is in a letter dated 23 June, 1836.

"Laplace, M. M. Plana et Damoiseau étaient sortis d'un principe faux, c'est à dire, qu'on pouvait rélativement à ces inégalités, les calculer prenant la longitude pour variable et changer ensuite v en nt, or cela se peut en effet pour le premier terme, mais cela n'est plus exact pour la seconde, de là toutes les erreurs où sont tombés les géométres relativement à ce terme, qui n'est pas le même dans leurs différents ouvrages, ni même le même dans le même auteur, lorsqu'il le determine, comme M. Plana, par deux méthodes différentes."

In my Theory of the Moon, I have calculated the terms in a great many of the lunar inequalities. I did not, however, endeavour to obtain the secular inequalities. The passage in vol. iii. p. 212 of the

Méc. Cél. appeared to me extremely obscure. I might, indeed, have studied the question in Plana's great work; but, notwithstanding the importance of Plana's results, his methods always have appeared to me so circuitous, that I have never had courage to follow them. Nor did I suspect any error in Plana's values of the secular inequalities; and I was greatly surprised when my friend Mr. Adams told me, that the term in the secular inequality for the longitude multiplied by m^4, given by Plana, was erroneous, and widely different from its true value.

Since my Part IV. was published, M. de Pontécoulant has completed his Theory of the Moon, carrying his approximations to the same extent as Plana, and leaving nothing to be done in order to enable his expressions to be employed in ephemerides, but the comparatively easy task of deducing tables from them. It is true, that perhaps the terms which he has taken from me and incorporated with his own, may be more considerable, both in number and magnitude, than the rest; but it must be recollected, that the terms which he has added to mine are those which are generally the most troublesome and difficult to obtain. Moreover, M. de Pontécoulant removed some numerical errors from my work, which, as I proceeded, would have infested the ulterior terms, and would have rendered the expressions inaccurate, even if complete. These corrections were duly acknowledged at the time; and I take this opportunity of repeating my acknowledgments to him of this mark of friendship, and for the pleasure and instruction I have derived from a correspondence continued for a period of nearly thirty years.

If M. de Pontécoulant's lunar theory has not been more duly appreciated, and if, like Prof. Hansen, he has not had the good fortune to obtain the gold medal of the Astronomical Society, I can only attribute it to the circumstance that, from the difficulty and complexity of the subject, very few are able to form an opinion upon his work.

My friend, M. de Pontécoulant, writes to me, in a letter dated the 14th inst.:—" Il est trés probable que votre méthode finira par prendre le dessus et être uniquement adoptée pour toute la théorie de la lune. M. Main finira par avoir raison et il aura predit seulement un peu plutôt ce qui arrivera dans la suite, mais vous auriez lieu de reclamen sur sa phrase (' some geometers, including M. Pontécoulant and Sir John Lubbock') car ce n'est point ainsi qu'on annonce si ce n'est une découverte du moins une *tentative aussi hardie* que celle de changer les méthodes usitées jusque là dans une théorie aussi ardue que celle de la lune et sans être aucunement sur de succés.........On est bien fort,

croyez moi, quand on se sert des formules que vous avez employées dans la theorie de la lune."

No one is more competent than my friend, to decide upon the advantages which my methods present over those previously in use; for no mathematician has practised them so extensively.

I cannot, indeed, be much gratified by the allusion which Mr. Main has made to me in his Address to the Astronomical Society (Monthly Notices).

Mr. Main says, "Some geometers (including M. Pontécoulant and Sir John Lubbock) have tried to solve the problem by using either the time or the mean longitude as the independent variable." This sentence appears to me to imply, that some geometers had paved the way in this direction, that M. de Pontécoulant followed in their wake, and that I brought up the rear.

If this is what Mr. Main intended, I can only say, that it is completely at variance with the truth. I began my Lunar theory, in which the developments are made at once in terms of the Moon's mean motion, without the support or advice of any mathematician, and without finding anything to my hand except the differential equations of motion.

The only mathematician who encouraged myself and my friend M. de Pontécoulant, was my lamented friend Poisson. He was well aware of the necessity which existed for an elaborate and careful verification of Plana's coefficients. But, unfortunately, he did all in his power to divert the attention of mathematicians from my methods. He mentions, indeed, my methods; but he recommended, in preference, the employment of the *variation of arbitrary constants*. This injudicious recommendation is given in a paper printed in vol. xiii. of the "Mémoires de l'Institut."

Poisson says:—"J'adopte d'abord l'idée des deux géomètres italiens, d'exprimer les coefficients des inégalités lunaires, en fonctions explicites des données de la question, qui pourront rester indéterminées dans la solution analytique. Mais je propose d'exprimer directement les trois coordonneès de la la lune, c'est-à-dire, sa longitude vraie, sa latitude et son rayon vecteur, en fonctions du temps, comme on le fait à l'egard des planètes, et comme M. Lubbock a déja entrepris de l'effectuer pour la lune, dans les derniers volumes des Transactions philosophiques et dans un écrit particulier. Je propose en outre de remplacer les équations différentielles relatives á ces trois coordonnées, par celles d'ou dépendent les six éléments elliptiques devenus variables, ou, autrement dit, d'employer dans le problême du mouvement de la lune, la méthode

de la *variation des constantes arbitraires*, dont j'ai précédemment montré l'usage dans la question du mouvement de la terre autour de son centre de gravité, et que l'on peut regarder, à juste titre, comme la plus générale et la plus féconde que les géomètres aient imaginée. J'explique, dans mon Mémoire, les avantages de ce double changement dans les méthodes ordinaires; après quoi j'examine successivement tous les points principaux du mouvement de la lune; et je montre, par des exemples choisis, comment on pourra appliquer à ce mouvement les formules connues de la variation des constantes arbitraires. A cette occasion, j'ai été conduit à m'occuper de nouveau du théorème sur l'invariabilité des grands axes et des moyens mouvements, que j'ai démontré, il y a vingt cinq ans, en ayant égard aux carrés et aux produits des forces perturbatrices. J'espére que les géomètres ne verront pas sans intérêt les développements que j'ai ajoutés à cette importante proposition, et l'application spéciale que j'en ai faite au mouvement de la lune."

The proof which Poisson gives in this paper, that the expression for the variation of the moon's axis major contains no argument of long period, accompanied by a multiple of m less than m^4, is incomplete. I succeeded in supplying the considerations which are wanting in my Part IV. p. 405. See M. de Pontécoulant's Théorie Anal. vol. iv. p. 416. According to Mr. Main (Address, p. 13), Poisson "deduces the lunar inequalities by the method of the variation of the elements, though the expansions are not completely effected." I consider this a very unfair representation of the facts. The truth is, Poisson pretends that the inequalities may be deduced in this manner; but there is no attempt to show how it can be carried out generally, and only a few terms of long period are considered.

This method may, as is well known, be used in determining the secular inequalities (it has recently been so employed by M. Delaunay) and the inequalities of long periods; but it is utterly impracticable in general; indeed, I am informed by M. de Pontécoulant, who had the advantage of personal intercourse with that consummate mathematician, that Poisson, before his death, fully recognised the superior advantages of my method. Poisson could not write legibly, and was no computer; the computations contained in his paper were done for him by M. Largeteau. The most interesting result obtained was, the showing that the coefficient of $\cos(3\tau - \xi + 3\xi' - 2\eta)$ in the lunar theory was equal to zero. I showed, in the *Phil. Trans.*, that this arose from the fact of the coefficient of $\cos(2\tau + 3\xi')$ being equal to zero; and in the same way, the coefficient of $\cos(2\tau + 2\xi')$ being equal to zero, the

coefficients of $\cos(2\tau+\xi+2\xi')$, $\cos(2\tau-\xi+2\xi')$, $\cos(2\tau+2\xi+2\xi')$, $\cos(2\tau-2\xi+2\xi')$, and $\cos(2\tau+2\xi'+2\eta)$, etc., are also equal to zero.

The only expressions in terms of the mean motion which (as far as my knowledge extends) existed when I began to work on the Lunar Theory, are those for r, v, and s, given by La Place in vol. i. of the Méc. Cél., p. 181.

If Mr. Main, by putting M. de Pontécoulant's name before mine, means to pretend that I took the idea from him, or was preceded by him in the publication of this method of treating the perturbations of the moon by the sun, he advances a pretension which would be repudiated by no one more strongly than by my friend, M. de Pontécoulant himself.

The first edition of my Part I. was published before; but the second edition has the date of the 15th Nov. 1834. The first mention of the moon in the correspondence of M. de Pontécoulant with me, occurs in a letter of his dated the 11th Sept., 1835, in which he writes :—"Je m'occupe beaucoup à ce moment, monsieur, de la théorie de la lune, et j'ai lu avec grand plaisir vos divers mémoires. J'approuve en son entier la marche que vous suivez dans le dernier (new edition) et la prefère de beaucoup à celle indiquée par M. Poisson que je regarde comme tout a fait impracticable, du moins quand on ne se borne pas à la recherche de quelque inegalité particulière."

And in a letter dated 13th Oct., 1840, he writes :—"L'ouvrage de Damoiseau est une œuvre consciencieuse, mais au dessous des autres parties de la théorie des inegalités planétaires, Plana avait le premier atteint le but veritable mais par une route pénible et herissée de difficultés. Enfin vous avez le premier imprimé des méthodes faites dans la vraie direction qu'il fallait suivre; je me suis empressé d'y entrer."

The date of my Part II. is April, 1836; of Part III., August, 1837; and of Part IV., October, 1840; so that my work on the Lunar Theory is the result of almost unremitting thought and labour during upwards of seven years; and I defy any one to produce a passage in the speech of the President of any scientific society, where an original work of the magnitude and importance of my Lunar Theory, has been so scurvily treated as mine has been by Mr. Main.

Neither M. de Pontécoulant nor myself can admit the sentence of Mr. Main, about the geometers, to be correct. We are not aware that M. Main can cite any geometer who before us treated the theory of the moon by the expressions in which the time is taken as the inde-

pendent variable. The Memoir of Poisson, which was subsequent to my first publications, is a theoretical enquiry, and, as Mr. Main observes, an interesting subject for study, but not a theory extensively applied. I therefore ask M. Main, in my name and in that of my friend M. de Pontécoulant, to retract or explain this assertion. My priority is incontestable; for the Theory of the Moon of M. de Pontécoulant only appeared in 1846; he waited to publish it, until it was complete: whereas I published my results by degrees, as soon as I obtained them. The intention of M. de Pontécoulant to treat the theory of the moon by a direct method, originated in conversations between him and Poisson, when M. Plana's work first appeared (Poisson told me this); and this idea was, I believe, strengthened, but not suggested, by my work.

I have a letter from M. de Pontécoulant, dated Sept. 4th, 1834, in which my much valued friend acknowledges the receipt from me of a letter referring to the Moon; and it appears that he was then occupied with the publication of vol. iii. of the Théorie Anal., which treats of the planetary inequalities. Afterwards, as I proceeded in my work, my friend favoured me with many numerical corrections, for which I feel greatly indebted, and all of which I have duly acknowledged. I regret that my friend did not pursue a similar course towards me. In the paper in the Conn. des Temps, to which I have referred above, the calculation of the coefficient of $\cos(2gt-2ct)$, p. 38, which is by no means easy (M. de Pontécoulant calls it "dur à arracher"), was done by me, and is not acknowledged to this day. Laplace found this coefficient equal to zero in the Conn. des Temps for 1824, p. 289. Plana, afterwards correcting Laplace in the Comptes rendus, gave another wrong value. M. de Pontécoulant, afterwards correcting Plana in the Comptes rendus, gave another wrong value. Plana afterwards corrected this error of M. de Pontécoulant, and, admitting his own, gave another wrong value, viz. $\frac{135}{32}$; the true value is $\frac{405}{128}$, which I first obtained, and sent to M. de Pontécoulant at his request, by whom it was inserted in the Conn. des Temps for 1840, p. 38.

And my own results are so interwoven with the author's in his fourth volume, that I hardly know which are mine, and which are not; nor can any one know which terms taken from me M. de Pontécoulant has taken the trouble to verify; for he treats all alike, merely saying, "on a formé l'expression suivante," or "nous avons trouvé" (pp. 100, 106, 121, 129, 162, and *passim*). I thought M. de Pontécoulant had

at least verified my development of R, because in p. 58 he expressly says, "J'ai formé l'expression suivante"; but the error which Mr. Cayley detected in my coefficient of *cos* τ (Theory of the Moon, p. 34), is to be found also in his work, vol. iv. p. 60, where the coefficient of *cos* ξ should be

$$+\frac{3}{8}\left(1+2e^2+2e'^2-\frac{11}{4}\gamma^2\right)$$

and not

$$+\frac{3}{8}\left(1+3e^2+3e'^2-\frac{11}{4}\gamma^2\right)$$

The same remark, according to Mr. Cayley, applies also to the terms of which the arguments are in my notation 2η, and $2\tau-2\eta$; and if they do not apply to others, it is, according to Mr. Cayley, because M. de Pontécoulant's development is not carried so far as mine.* See the Memoirs of the Ast. Soc. This development is now, thanks to Mr. Cayley, irrefragably established.

A paper by Mr. Cayley is mentioned by Mr. Main: this paper contains a verification of my development of the disturbing function, which occupies less than six pages of my work.

I quote the following extraordinary passage from Mr. Main's Address *verbatim*,† lest I should be accused of misrepresentation:—"I wish, in particular, to point out to you, that the imperfect state of the lunar theory (or, rather, of its embodiment in tables, for Plana's great work even then existed) had attracted attention. Sir John Lubbock was then engaged in the attempt, to which I have before alluded, of applying a totally new method to the lunar theory, by making the mean longitude the independent variable in the equations of motion; and, in the course of his researches, had frequent correspondence, involving many deep questions, with Mr. Airy; and the chief difficulty which

* "The greater part of the discordant terms do not occur in Pontécoulant's development, which is not carried so far; and the only differences which I find in the coefficients of Pontécoulant's $R(=\Omega)$, are as regards the arguments 18, 57, 70, corresponding respectively to Lubbock's arguments, 62, 63, 101, included in the preceding table, and for which Pontécoulant's coefficients, correcting for the change of sign, correspond with those given by Lubbock."—Cayley on the Development of the disturbing Function in the Lunar Theory.

† Address of the President of the Royal Astronomical Society, the Rev. Robert Main, M.A., on the presentation of the gold medal to Prof. Hansen, for his Lunar Tables, on Feb. 10th, 1860 (p. 14).

they had to encounter, as I before hinted, was the disentangling of the artificial arguments devised by Burckhardt, and the finding out what values of the coefficients, and what arguments as deduced from theory, he really had used, so as to enable Sir John to compare his own results with them. A good deal of labour was fruitlessly bestowed on this part of the subject." What can have induced M. Main to make so gross a misstatement, or rather so many misstatements, I am at a loss to imagine.

I beg to repudiate the compliment kindly intended for me by Mr. Main, and to state, that I never attempted to make the mean longitude the independent variable in the equations of motion, seeing that this had been accomplished more than half a century before I was born.

The opinion, that the imperfect state of the embodiment of the lunar theory in tables had attracted attention, is singular, and probably peculiar to Mr. Main; for if the Astronomer Royal had been of the same opinion, how easy for him, with the ample data furnished by Plana, to put a computer to embody Plana's expressions in tables. As regards myself, it was not the imperfect state of the embodiment in tables that attracted my attention, but the imperfect state of the theory itself. Plana's method appeared to me then, as it does now, unnecessarily difficult, circuitous, and prolix. And, moreover, what confidence could astronomers place in such gigantic calculations, when, as far as I know, not a single term had been verified? And would any one think of embodying such expressions in tables, and using such tables for the purposes of navigation, without being able to form any idea how many of the terms could be relied on as accurate? Now the case is different: M. de Pontécoulant and I, partially also Mr. Adams, are able to testify, that the care with which M. Plana conducted his great work, and the accuracy of his numerical calculations, are marvellous and beyond all praise; and we can safely affirm, that tables founded upon his expressions, with such corrections as my researches and those of M. de Pontécoulant and Mr. Adams have elicited, and with Airy's constants deduced from the Greenwich observations, would furnish places as near or nearer to observation than any other.

I hope for Mr. Main's support in this view of the question; but the Astronomer Royal no doubt believed M. Plana's expressions to be full of mistakes, notwithstanding the verification they have undergone; or if not, why did he abandon those which all astronomers can understand and appreciate, and betake himself to Prof. Hansen's, which no mathematician has seen or verified. This step of the Astronomer

Royal, if approved by astronomers and if persisted in, is equivalent to ignoring all that has been done by Plana, by M. de Pontécoulant, and by myself; to say nothing of Laplace, Damoiseau, and all the great mathematicians of the last century. As long as I have a voice, I will endeavour to make it heard against this act of barbarity, and I will contend for the necessity of a literal solution first insisted upon by the Astronomer of Turin, even against so powerful an adversary as the Astronomer Royal.

> *Οὔτις, ἐμεῦ ζῶντος καὶ ἐπὶ χθονὶ δερκομένοιο,*
> *Σοὶ κοίλῃς παρὰ νηυσῖ βαρείας χεῖρας ἐποίσει*
> *Συμπάντων Δαναῶν· οὐδ' ἢν 'Αγαμέμνονα εἴπῃς,*
> *῝Ος νῦν πολλὸν ἄριστος 'Αχαιῶν εὔχεται εἶναι.*

But, in fact, Tables of the Moon were published in America in 1853, founded apparently on the values of the coefficients given by M. de Pontécoulant in vol. iv. p. 624. These tables are used for finding the places of the moon given in the American Ephemeris; and if the comparisons with observation given in the Ast. Nach. 1255, in the American Astronomical Journal, vol. vi. No. 9, and in the Monthly Notices for May 11th, 1860, can be depended upon, their accuracy is scarcely, if at all, inferior to Hansen's tables. Still I feel sure they might be improved; but even if those tables, founded upon intelligible and literal expressions, were less accurate, it would be easy, by a careful discussion and examination of the differences between calculated and observed places, to discover which coefficient or coefficients wanted retouching; and no doubt, in many cases, the approximations ought to be carried further. If, after all, the tables failed to satisfy the observations, as Mr. Adams's secular inequality of the longitude appears to do, we should have an indication of great value, not that the tables were wrong, but that some other principle in the mechanism of the heavens remained undetected. So Prof. Hansen, finding the places given by his tables differ from observation, was led to his wonderful discovery, that the centre of gravity of the moon differs from her centre of figure. See letter of Prof. Hansen to the Astronomer Royal, published in the "Monthly Notices" for Nov. 10th, 1854. It would be interesting to know whether this new principle, for which we are indebted to the errors of Prof. Hansen's coefficients, extends to the earth and the other planets, or whether it is confined to their satellites. It is due to Prof. Hansen to remark that, in p. 570 of the 2nd. vol. of the Méc. Cél., Laplace appears to have proved that the moon is not homogeneous, and that she has not the form which she would have had, if she had primitively been

in a fluid state. But as far as I am aware, there is nothing in the Méc. Cél. to lead to the supposition that the centre of gravity of the moon does not coincide with her centre of figure.

That it would be desirable to carry the approximations further, is evident from the fact that M. de Pontécoulant has added the following terms, by mere induction, to his expression for the longitude, in vol. iv. p. 601.

$$\underset{[3]}{-\cdot550\sin(2\tau-\xi)}+\underset{[9]}{\cdot500\sin(2\tau-2\xi)}+\underset{[11]}{\cdot500\sin(\xi+\xi_{,})}+\underset{[14]}{\cdot720\sin(\xi-\xi_{,})}$$

$$\underset{[15]}{+1\cdot45\sin(2\tau-\xi+\xi_{,})}+\underset{[21]}{\cdot500\sin(2\tau-3\xi)}-\underset{[23]}{\cdot590\sin(2\xi+\xi_{,})}+\underset{[24]}{\cdot450\sin(2\tau-2\xi-\xi_{,})}$$

$$\underset{[26]}{+\cdot650\sin(2\xi-\xi_{,})}+\underset{[32]}{\cdot220\sin(\xi-2\xi_{,})}-\underset{[64]}{\cdot600\sin(2\tau+2\eta)}+\underset{[65]}{\cdot180\sin(\xi-2\eta)}$$

$$\underset{[101]}{-\cdot700\sin\tau}-\underset{[102]}{\cdot400\sin(\tau-\xi)}-\underset{[103]}{\cdot160\sin(\tau+\xi)}+\underset{[105]}{\cdot300\sin(\tau+\xi_{,})}+\underset{[131]}{\cdot700\sin4\tau}$$

$$\underset{[132]}{+\cdot500\sin(4\tau-\xi_{,})}$$

M. de Pontécoulant has added, by induction, the following terms to his expression for the latitude (vol. iv. p. 611).

$$\underset{[148]}{\cdot600\sin(2\tau+\eta)}+\underset{[151]}{\cdot500\sin(2\tau-\xi-\eta)}+\underset{[155]}{\cdot800\sin(\xi_{,}-\eta)}+\underset{[156]}{\cdot800\sin(\xi_{,}+\eta)}$$

$$\underset{[169]}{+\cdot500\sin(2\tau-\xi-\xi_{,}-\eta)}-\cdot200\sin3\eta+\cdot85\sin(4\tau-\xi-\eta)$$

We have the authority of the Astronomer Royal for believing that tables founded on Plana's coefficients would be far preferable to Burckhardt's. The Astronomer Royal, in the *Monthly Notices* for April, 1859, says, "It must be borne in mind, that no comparison has been made either of Damoiseau's tables, or of tables formed from Plana's theory, adapted to corrected values of the elliptic constants; and that it is not improbable that they might be found to occupy a very high place as regards accuracy. The comparisons in the *Greenwich Lunar Reductions* of Plana's theory, based on Damoiseau's and Plana's elliptic constants with observations, show that it is greatly superior to Burckhardt's."

Mr. Main states, that the chief difficulty we had to encounter (i.e. the Astronomer Royal and myself), in applying a totally new method to the lunar theory, by making the mean longitude the independent variable in the equations of motion, was the disentangling of the artificial arguments devised by Burckhardt, and the finding out what value of the coefficients, and what arguments as deduced from theory, he

really had used, so as to enable me to compare my own results with them.

It is not true that the Astronomer Royal attempted to apply a new method to the lunar theory; and I do not recollect having received a useful hint of any kind, or a correction (for so many of which I am indebted to M. de Pontécoulant) from him. I do not make this a reproach to the Astronomer Royal: he has revolutionised the theory and the practice of the construction of astronomical instruments, as well as of the making and ordering of observations; he has borne the chief labour in almost every government commission for scientific purposes; he has been engaged in optical researches, or in writing profound memoirs on some branch of abstract or mixed science, some of which are in the *Transactions* of the Astronomical Society: his labours in other branches of astronomy have been so important, and the occupations of his high position so incessant, that he may well be excused if, in addition to his other contributions to science, he has not attempted to calculate from theory any of the lunar irregularities. But so far from his ever having given me the slightest assistance or encouragement in that direction, his views and opinions on the lunar theory appear to be opposed to mine, and in accordance with those of Prof. Hansen.

I fear that there must be something in my correspondence with the Astronomer Royal, which has led M. Main to think that I wanted to compare my results with Burckhardt's; in which case I must admit myself in error; for my expressions had not then, nor have they since, arrived at that degree of completeness, without which they could not (with very few exceptions) be comparable with those of Burckhardt properly transformed. It is true, that I wished to compare his coefficients with those of Plana and Damoiseau; for as, to my great regret, Burckhardt's tables were then used for the Nautical Almanac, and as the Greenwich observations were compared with the places from Burckhardt's tables, it was in this way possible to form a rude idea of the accuracy which places calculated by means of Plana's or Damoiseau's expressions would attain. I was very much surprised to find how nearly all the three coefficients agreed. But I do not consider this the most important object which can be achieved by a transformation of Burckhardt's tables, if performed with sufficient care. Suppose Burckhardt's tables to be used to procure places of the moon, and suppose it should be wished to get some places from Plana's values of the inequalities, it will not be necessary to throw all his inequalities into tables, but only their differences from my transformed

Burckhardt's coefficients, which will be sufficient to give the differences between those places and those of Burckhardt. And so for M. de Pontécoulant's or any other. It is not true, therefore, that labour was fruitlessly bestowed upon this part of the subject.

It is true, that I entreated the Astronomer Royal to give me his assistance in transforming Burckhardt's expressions; for although I was conversant with the lunar theory generally, this calculation is not exactly the same as that of the determination of a coefficient in the perturbations of the moon, and it is extremely troublesome. In consequence of his valuable aid, I believe the transformation of Burckhardt's expression for the longitude, given in p. 331 of my work, may be confidently relied upon. Unfortunately, the Astronomer Royal was unable to assist me in the transformation of the latitude; and it was still more unfortunate, that we both omitted to transform Burckhardt's expression for the lunar parallax. We thought those errors impossible, which were afterwards detected by the sagacity of Mr. Adams; and which could not have escaped us, if we had treated Burckhardt's expression for the parallax as the Astronomer Royal and myself treated his expression for the longitude, and as I treated his expression for the latitude. This unfortunate omission, as is well known, retarded the reduction of the Greenwich observations of the moon, and made it necessary to do over again a great deal of that work, entailing much additional labour and expense upon the Admiralty.

Professor Hansen has propounded lunar tables which are characterised by two peculiarities.

In the first place, no one, if I am correctly informed, has seen the expressions upon which they are founded; and probably, if they did, not a dozen mathematicians in Europe would understand them.

In the next place, his expressions are not in the same form as those of any other mathematician.

The consequence of these peculiarities is this, that it is utterly impossible to form any notion of the accuracy of Prof. Hansen's expressions, except by that test, which is so satisfactory to the Astronomer Royal and to Mr. Main, the test of the comparison of places obtained by theory with those obtained by observation.

Mr. Main admits that advantages must be allowed to be attached to the preservation of the literal values of the coefficients throughout the whole investigation, as was first done by Plana, and is the plan pursued by Laplace latterly, by Poisson, by M. de Pontécoulant, by Mr. Adams, by Delaunay, and by myself; in fact, by every mathematician except Prof. Hansen; but I go further, and I say, that, in my opinion, any

theory which does not do so, in the present state of the subject, is of little use.

The opinions of M. Plana, which the Astronomer Royal and Prof. Hansen disregard, are recorded in the following passages of his great work:—

"En général toute théorie de la lune qui ne fournit pas précisément ces coefficiens numériques absolus, doit être regardée comme incomplète, puisque ces nombres aussi mathématiquement exacts que ceux qui entrent, par exemple, dans la serie de Taylor, sont une conséquence nécessaire de la gravitation universelle et de la forme des équations différentielles qu'il s'agit d'intégrer" (Plana, vol. i. p. 142).

"Les théories de la Lune publieés jusqu'ici n'offrent pas une expression litterale et explicite des trois coordonnées: elles portent toutes le caractère d'une solution qu'on pourrait appeller mixte, en réfléchissant qu'on y procède par des opérations algébriques entrelacées avec des operations arithmétiques; or les quantités numériques absolues se trouvent enveloppées avec les valeurs spéciales des constantes arbitraires. Outre cela, on ne peut pas dire, à la rigueur, que l'approximation ait été conduite de manière, que les coefficiens y sont exacts dans les quantités d'un ordre déterminé; du moins au de là du troisième ordre, généralement parlant. Les preuves de cette assertion se déduisent des résultats mêmes de ces théories, en les ramenant à la forme précise que nous avons adoptée. Alors on voit souvent des differences entre les coefficiens numériques absolus qui disparaissent par un examen fait avec soin sur les différentes parties qui concourent à leur formation; et cela, en dévoilant l'omission de celles résultantes du développement de quelques termes du même ordre et de la même forme que ceux qu'on a conservés. Nous avons donné plusieurs exemples de comparaison de cette espèce dans le cours de cet ouvrage; et il faut en examiner les details pour se persuader de leur justesse. Notre but, dans ces comparaisons, a été de faire voir seulement, que le principe, par sa nature inviolable, de tenir compte de toutes les quantités du même ordre n'a pas été toujours suivi; et que par fois, on s'en est affranchi en commettant l'inconsequence de conserver des termes plus petits que ceux que l'on avait négligés. Mais nous admettons, que, par des heureuses compensations, on soit tombé sur des résultats sensiblement d'accord avec l'observation. Car on sait bien, que les erreurs de ce genre seront considerablement atténuées après la réduction en nombres des facteurs qui conservent naturellement la forme littérale jusqu'à la fin de l'integration. Mais cela n'empêche pas, que leur existence ne soit un motif suffisant pour rendre la solution plus ou

moins contraire à la doctrine des séries, et faire attribuer le succès à des relations fortuites entre les grandeurs données par l'observation" (Plana, Introduction, vol. I. p. viii.).

"En s'écartant de ce principe, il est impossible que les coefficiens cherchés soient exacts, analytiquement parlant; il est possible qu'ils le soient, plus ou moins, aprés la réduction en nombres des facteurs qui conservent la forme littérale. Mais dans l'état actuel de la science on est en droit d'exiger une détermination des différens coefficiens qui, dans un ordre déterminé, soit exacte à l'egard des coefficiens numériques absolus, dont la recherche constitue la véritable difficulté du problème" (Plana, vol. i. p. 157).

I have quoted these passages from M. Plana's work, because I think them of great importance; and because I think, if they had been more generally known to astronomers, the Astronomer Royal would not have ventured to support Prof. Hansen's views in the opposite direction, or have contributed the money of the government to bring forward his tables.

It is quite true, that, by ignoring this principle, expressions can be obtained with greater facility; and both myself and M. de Pontécoulant wasted some time originally in this direction; but this facility is obtained by the sacrifice of all means of detecting error.

Not only M. de Pontécoulant and myself were aware of the facility which may be obtained by abandoning the literal development, but after he had made considerable progress in the subject, he was almost tempted to give it up, as appears by the following passage in a letter without date, which I received from him probably in 1838:—"Je suis effrayé de voir ce qui reste encore à faire pour atteindre le même degré de precision que M. Plana et pourtant tant que nous n'avons pas rempli cette tâche on nous dira que notre travail est incomplet, c'est là ce qui m'a fait regretter un moment de ne pas avoir adopté une marche analogue a celle de Laplace, la solution est numérique il est vrai, mais elle peut etre completé sans de trop grands efforts ce qui est un avantage immense et que n'offre pas la solution littérale qui se complique de plus en plus à mesure qu'on avance."

In fact, there are four methods which have been used with a view of procuring places of the moon sufficiently good to be used for finding the longitude at sea.

First. In the infancy of the art, empirical tables were deduced from observation, the arguments of the equations being those suggested by theory.

Secondly. Tables were deduced from the theory of Laplace by a solution more of an arithmetical, than a symbolical nature; such are the Tables of Damoiseau.

Thirdly. Plana produced his analytical or symbolical method, in which the coefficients are given literally, but deduced from the inverse expressions by means of the reversion of the series.

Lastly. I published the direct solution, which M. de Pontécoulant has worked out, in which the coefficients are given in a literal form; and the differential equations employed are those used in determining the perturbations of the planets. When the approximations shall have been carried to a sufficient extent, if that has not been done already, I do not see how any further improvement will be possible.

Prof. Hansen's method deviates from all of these, inasmuch as he finds a quantity which being added to the mean anomaly in the elliptic expression for the longitude, gives the true value of the longitude, including the perturbations caused by the sun.

Of course, Prof. Hansen's places are not wide from the mark, especially as he alters his coefficients, to make them agree with observation, by supposing the centre of gravity not to coincide with the centre of figure of the moon. Probably not one of our coefficients is much in error; but there is this difference: every mathematician may, if he chooses, contribute something to improve the latter; they have been improved already, and they will be gradually improved more and more, and brought up to keep pace with the requirements of navigation and the observatory in future ages: whereas no one can judge of the value of Prof. Hansen's intrinsically, and no one has any chance of detecting or removing any mistake, if such should exist.

Professor Hansen professes to have constructed his tables according to the Newtonian principle of universal gravitation. Let us see whether they are entitled to this character.

M. Hansen assumes, that the centre of gravity of the moon does not coincide with her centre of figure, and that, in consequence, all the coefficients of perturbation for the mean longitude must be multiplied by a constant factor, and his latitudes diminished by 1″; this factor, which he found from the errors of his coefficients to be 1·0001544 *

* "Le facteur des inégalités de l'anomalie moyenne resultant de la distance entre le centre de gravité de le lune et le centre de sa figure est

= 1·0001544

for some, and 1·03573 for others, enabled him to augment his erroneous value of the evection by ″·69 ; and he reminds the Astronomer Royal that he found Plana's value of the coefficient of the evection to require an increase of 1″·28, the value of his coefficient of the variation to require an increase of ″·68, and of the coefficient of the annual equation of 1″·07. But M. de Pontécoulant, going over the ground after Plana, found the values which are given in p. 27, agreeing exactly with the determinations of the Astronomer Royal. If this new theorem can be proved, our coefficients, which are now so close to those for which astronomy is indebted to the Astronomer Royal, must be multiplied by 1·0001544. This would raise our value of the coefficient of evection to 4587″·711, and our coefficient of variation to 2371″·16, making the errors or differences from the values determined by the Astronomer Royal respectively ·″90 and ·″37, instead of ·011 and ·099. This successful determination of the evection by M. de Pontécoulant, includes ·″55 obtained by mere induction (see p. 16); and the result is, therefore, less satisfactory than it would have been if the coefficient had been entirely found by calculation. According to the last determination of the Astronomer Royal, the coefficient of the parallactic inequality is − 124″·7. In order to procure a coincidence with this value and that of M. de Pontécoulant, which is —122″·378, it would only be necessary to raise the value of the sun's parallax from 8″·6, the value employed by M. de Pontécoulant, to 8″·763, an alteration which is admissible. But to raise this coefficient 4″·3, as Prof. Hansen does by the arbitrary factor 1·03573, would require an alteration of the sun's parallax by ·″23. I consider it impossible by Prof. Hansen's methods, or by any numerical solution, to be certain of arriving at great accuracy in the values of the coefficients; but in the hands of so great a mathematician as Prof. Hansen, and so very skilful a computer as Prof. Hansen, I should not anticipate in any coefficient such an error as 4″·3. Until Prof. Hansen is able to derive this theorem, that the moon's centre of gravity does not coincide with her centre of figure, from the laws of gravitation, we shall maintain that our coefficients of longitude do not want to be multiplied by any factor,

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le facteur indiqué par les observations, par lequel ces inégalités précédentes, qui dependent de la parallaxe du soleil, doivent être multipliées, est

1·03573

et à cause de la coordonné perpendiculaire à l'écliptique du centre de figure de la lune par rapport à son centre de gravité, aux latitudes données par les expressions précédentes il faut ajouter — 1″·00." (Introduction to Professor Hansen's Tables, p. 16.)

or our latitudes to be diminished by $1''$; we shall maintain that our coefficients are correct, and that Prof. Hansen's are at best empirical, and not deduced, as he pretends, from the Newtonian law of universal gravitation.

But if Prof. Hansen moves the moon's centre of figure ever so far from her centre of gravity, will he by so doing cure his erroneous value of the evection? I fear not; and I think he will have to doctor it upon some other principle. I trust Prof. Hansen will give up the Newtonian law of universal gravitation, and admit that his tables only pretend to be empirical.

But it may strike the reader as possible, that this surprising agreement between our coefficients and those of the Astronomer Royal may arise from the circumstance that M. de Pontécoulant, instead of having calculated the coefficients as he pretends to have done, copied them from the Greenwich Reductions. I may therefore mention, that the determinations of the Astronomer Royal were first published in 1848; whereas M. de Pontécoulant's Lunar Theory was published before, viz., in 1846. The Astronomer Royal's determinations, quoted above, are not yet published; and he has kindly given me a proof of his paper read at the Astronomical Society on the 8th July, 1859, from which they are taken; so that M. de Pontécoulant has not yet seen them.

Mr. Main says (Address, p. 13):—

"There is still one more work of considerable importance, namely, the American *Tables of the Moon*, published in 1853, under the superintendence of Lieutenant Davis, U.S.N., and by the authority of the Secretary of the Navy, according to the arrangement of the distinguished mathematician Professor Peirce. These Tables are based on Plana's theory; but the coefficients of the inequalities are corrected by the results given by the Astronomer Royal as arising from the discussion which closed his great work of the Reduction of the ancient *Greenwich Observations*, as well as by some corrections due to investigations of Professor Longstreth. There are also included, Hansen's two long inequalities due to the action of *Venus*, as well as his value of the secular equation of the moon's mean motion, and of the motion of the perigee. If these Tables be considered merely with reference to their current value (that is, as giving tabular places of the moon worthy of considerable confidence), there can be no dispute with regard to the high estimation in which they should be held; but the want of uniformity in the chosen values of the coefficients of the inequalities, must detract in some measure from their value, when considered in relation only to the advancement of the abstract theory of the lunar motions."

I entirely acquit Mr. Main of making any intentional misstatement, either as concerns these American Tables or any other subject noticed by him in his Address. I am confident his surprise would not have been less than mine, if he had found out the true character of these American Tables, or that eight of the empirical corrections of Mr. Longstreth, published by him in 1853, had been obtained from theory with immense labour, and published by M. de Pontécoulant seven years previously. But, unfortunately for M. de Pontécoulant and myself, Mr. Main's statements do us as much mischief as if they had been made intentionally. Mr. Main's high character as an astronomer, his professional position at the Greenwich Observatory, and his distinguished position as President of a Society which has contributed as much as any other in this country to the advancement of science, give his opinions a weight which mine, who am only a banker and an amateur, will not carry with them; and hundreds will read his Address, who will never read these pages.

The following is the first sentence in the Introduction to the American Tables :—

"These tables are constructed from the theory of PLANA, modified by the theoretical investigations of HANSEN, and the empirical corrections of AIRY and LONGSTRETH."

The plain meaning of this sentence is, that they had availed of the expressions used by Plana generally, and that they had obtained the coefficients independently, or that Hansen had modified them; but it is not so. The coefficients of Plana, verified and confirmed by Pontécoulant and myself, in the expression for the longitude, are adopted universally, except in eleven cases where empirical corrections of Plana's coefficients were furnished by Mr. Longstreth, and in seven cases where the coefficients had been determined empirically by Airy. But even so, justice is not rendered to our work; for the coefficients are only given to tenths of seconds, although we obtained them to the hundredths and even thousandths of seconds: as this is the case, a great deal of our work is thrown away. This degree of accuracy is, moreover, inconsistent with the extent to which the decimal places are carried in the periods of the arguments which are given in some cases, as in that of the variation, to the ten-millionth of a day, ·00001″ in sexagesimal seconds, or the hundred-thousandth of a second.

Eight of the empirical corrections of Mr. Longstreth correspond to those arguments to which my friend, M. de Pontécoulant, subjoined the following note :—"Nous avons marqué d'un (*a*) les resultats de M. Plana que nous avons reconnus fautifs par la revision de ses formules."

Mr. Longstreth does not tell us what observations he made use of to obtain these empirical corrections, or how he set about it.

There are eight empirical corrections of Mr. Longstreth in the American Tables, corresponding to the following arguments:—

Lubbock.		The Americans.		Coefficient.		
Index.	Argument.	Index.	Argument.	Plana.	Pontécoulant.	Longstreth.
7	$2\tau+\xi_{,}$	15	[120]	$-$ 23·611	$+$ 25·036	25·0
11	$\xi+\xi_{,}$	10	[101]	$-$111·099	$-$109·886	$-$110·0
12	$2\tau-\xi-\xi_{,}$	5	[1′21′]	$+$209·742	$+$206·919	$+$206·9
24	$2\tau-2\xi-\xi_{,}$	21	[12′2]	$+$ 7·762	$+$ 9·161	$+$ 9·2
27	$2\tau-2\xi+\xi_{,}$	69	[1′2′2]	$-$ 1·395	$-$ 1·277	$-$ 2·2
133	$4\tau+\xi$	26	[41]	$+$ ·855	$+$ 1·277	$+$ 1·9
136	$4\tau-2\xi$	7	[42′]	$+$ 34·518	$+$ 31·153	$+$ 31·2
	$4\tau-2\xi-\xi_{,}$	32	[1′42′]	$+$ 1·197	$+$ 1·191	$+$ 3·0

It appears, from the Appendix to the American *Nautical Almanac* for 1855, that in addition to the above, Mr. Longstreth furnished the following three corrections. Nothing is there said of the mode adopted by Mr. Longstreth to find these corrections, except that he obtained them empirically, which is very unsatisfactory.

Index.	Argument.	Plana.	Pontécoulant.	Longstreth.
64	$2\tau+2\eta$	$-$ 3·376	$-$ 5·728	$-$ 5·776
65	$\xi-2\eta$	37·191	39·427	39·391
67	$2\tau-\xi-2\eta$	·030	·514	·530

Of these eleven coefficients furnished by Mr. Longstreth, eight are identical with those of M. de Pontécoulant; of the remaining three coefficients, which therefore require particular attention, the following are the values according to

		Bürg.	Burckhdt.	Damois.	Plana.	Pont.	Longsth.
27	$2\tau-2\xi+\xi_{,}$	2·5	2·4	2·5	$-$1·4	$-$1·3	2·2
133	$4\tau+\xi$	1·9	2·3	·43	·85	1·3	1·9
	$4\tau-2\xi-\xi_{,}$		2·3	3·0	1·2	1·2	3·0

The weight of observation appears decidedly against our values; but it will be seen hereafter how difficult it is to suppose Pontécoulant's values erroneous.

There are two other cases where Pontécoulant says that Plana's coefficients are in error: in the one, the Americans adhere to Plana's value; in the other, they appear simply to have taken Pontécoulant's value, and called it Plana's. The following are the cases in question:—

Lubbock.		Coefficient.		
Index.	Argument.	Plana.	Pontécoulant.	The Americans.
14	$\xi - \xi_{,}$	+148·059	+147·691	+148·1
22	$2\tau + 3\xi$	+ 3·309	+ ·902	+ ·8

It is usual for mathematicians and philosophers working upon the same subject, even when they arrive at contrary conclusions, to present each other with copies of their works; but I only first heard of the existence of these American tables from the sentence in Mr. Main's address, and neither M. Plana nor M. de Pontécoulant have seen them to this day.

There are eleven coefficients in the longitude, in which Pontécoulant differs from Plana, and says that he has traced the mistake; "reconnus fautifs," is the expression of Pontécoulant.

		Plana.	Pontécoulant.	The Americans.	
7	$2\tau + \xi_{,}$	— 23·611	— 25·036	— 25·0	L.
11	$\xi + \xi_{,}$	—111·099	—100·886	—110·0	L.
12	$2\tau - \xi - \xi_{,}$	209·742	206·919	206·9	L.
14	$\xi - \xi_{,}$	148·059	147·691	148·1	P.
22	$2\tau + 3\xi$	3·309	·902	·8	P.
24	$2\tau - 2\xi - \xi_{,}$	7·762	9·161	9·2	L.
39	$2\tau - 4\xi$	·873	·606	·9	P.
64	$2\tau + 2\eta$	— 3·376	— 5·728	— 5·8	L.
65	$\xi - 2\eta$	37·191	39·427	39·4	L.
133	$4\tau + \xi$	·855	1·277	1·9	L.
136	$4\tau - 2\xi$	34·518	31·153	31·2	L.

Of these eleven corrections, published by Pontécoulant in 1846, Mr. Longstreth pretends to have discovered empirically eight in 1853; but, in the Introduction to the American Tables, no mention is made either of myself or of Pontécoulant. The Americans appear to have taken Pontécoulant's coefficient (Arg. 22), slightly altering it, and attributing it to Plana.

The American Tables were published in 1853, and have been used since then for the calculation of the places in the American ephemeris. M. de Pontécoulant's fourth volume was published in 1846. The places given by the American Tables appear to have an average error of about $2''{\cdot}5$; therefore they are within the errors of observation. A just posterity will give, not to the Americans who employed our coefficients, not to Prof. Hansen who published empirical tables in 1857, but to us, that is, to Plana, Pontécoulant, and Lubbock, who in 1846 furnished the means of constructing tables of the moon without any empirical hypothesis, the credit of first bringing the errors of the lunar theory within the limits of the errors of observation.

I regret extremely to differ from so great a mathematician as Prof. Hansen, upon the merits of an empirical or even a numerical solution in the lunar theory; but it would be unjust to deny to Prof. Hansen the very great importance of his discovery of the inequalities of the moon due to Venus; and should he succeed in determining their coefficients from theory, no mathematician will be more glad than myself to welcome the triumph of the illustrious astronomer of Gotha.

The Astronomer Royal's most recent determinations are as follows:

	$''$	$''$	$''$
Variation . . .	2370·7	Ours are 2370·199	Diff. ·099
Evection . . .	4587·01	„ 4586·999	„ ·011
Annual Eq. . .	669·0	„ 668·932	„ ·068
Par. Eq. . . .	124·37	„ 122·371	„ 1·992

But if we take the Astronomer Royal's determination of the Par. inequality from the meridional observations, it is $122''{\cdot}79$, and the difference from ours is only $''{\cdot}412$. The equation of the centre, according to the Astronomer Royal, is $22639''{\cdot}06$.

The notation of these American Tables is so curious, that for some time I could not identify the arguments; and I should never have done so, if my friend Mr. Adams had not kindly enabled me to decipher them. This notation is unlike anything I ever met with. The following comparison with my notation may be useful to others who have the same difficulty as myself:—

Lubbock.		The Americans.	
Index.	Argument.	Index.	Argument.
1	2τ	3	[20]
2	ξ	1	[1]
3	$2\tau-\xi$	2	[21̄]
4	$2\tau+\xi$	7	[21]
5	$\xi_{,}$	4	[100]
6	$2\tau-\xi_{,}$	8	[1̄20]
7	$2\tau+\xi_{,}$	15	[120]

The Americans express the arguments by numbers, "in which the digit occupying the place of units is the coefficient of x, that in the place of tens is the coefficient of t, that in the place of hundreds is the coefficient of z, that in the place of thousands is the coefficient of y, and that in the place of tens of thousands is the coefficient of u." The

accents upon the numbers indicate that the coefficient is negative. Thus the number of the argument [14′253′] denotes

$$u-4y+2z+5t-3x.$$

z is the mean anomaly of the sun.
u the uncorrected mean longitude of the moon.
x the mean anomaly of the moon.
y the mean argument of the latitude.
t the argument of the variation.

The Americans also use the same index for any argument and for its multiples; the parallactic equation and the variation thus have the same index. This method may do as far as tables are concerned, but of course could not be used in the mathematical theory. The indices are also arranged in the order of the magnitude of the coefficients; they differ, therefore, entirely from mine and from those of M. de Pontécoulant.

Nor is it true, as Mr. Main states, that the coefficients of the inequalities are corrected by the results given by the Astronomer Royal; the fact is, in the expression for the longitude, 7 coefficients are taken from Airy's work, as determined by him from the observations empirirically: all the rest are taken from us; that is, Plana, Pontécoulant, and myself.

As these American Tables are based on theory, and as, according to Mr. Main, they are worthy of high estimation, why were they not used in the *Nautical Almanac?* and why were they not taken as the standard of comparison with Prof. Hansen's tables by the Astronomer Royal, in April, 1859, instead of Burckhardt's tables, which were published in 1812, that is, more than forty-seven years ago? The Astronomer Royal says, that "probably in no recorded instance has practical science ever advanced so far in accuracy by a single stride." This misrepresentation of the real facts of the case, is arrived at by ignoring altogether the American Tables and the labours of Plana, Pontécoulant, and myself, by whom the coefficients employed in the American Tables were obtained. If the Astronomer Royal had compared Hansen's tables with those of D'Alembert, published in 1754, the stride might have been made to appear much greater Nor was the Astronomer Royal ignorant of the fact that Plana's theory was greatly superior to Burckhardt's, as may be seen from the passage I have quoted from the Astronomer Royal in p. 12.

Upon one point I thoroughly agree with Mr. Main. Mr. Main laments, that in the *Nautical Almanack*, and other similar works,

Damoiseau's Tables had not been chosen, instead of those of Burckhardt. I not only lamented it, I remonstrated with the late superintendent on the subject; but I did not carry my remonstrances far, as I was painfully aware that my opinions carried no weight with him. Mr. Stratford discharged the duties of his position very ably and conscientiously, and he raised our national Ephemeris to a degree of completeness which it never had attained before, and which no other similar work has yet attained (as far as I know); but he had not the knowledge of theoretical astronomy, which would have enabled him to understand the merits of a question of this kind.

My opinion on this subject is the same now as it has always been. I think all empirical tables are an opprobrium, and are only to be tolerated as long as none others can be found, except such as are considerably worse.

I estimate the average error of Burckhardt's tables to be 8″ of space; that of the American Tables to be 2·5″; and that of Hansen's tables to be 2″. If this estimate be correct, and I am certain it is not far from the truth, if we take the stride made by Hansen for unity of stride, that made by the American Tables is represented by eleven; and therefore, so far from the fact being as stated by the Astronomer Royal, that no practical science was ever advanced so much by a single stride as it was by Hansen's tables, we see that in the very same subject the stride made by us whose labour contributed to form the American Tables, is eleven times greater than that made by Hansen.

Unless energetic steps are taken to obtain tables for calculating the place of the moon from our coefficients, the Astronomer Royal's injudicious patronage of Prof. Hansen's methods may retard the advancement of the lunar theory by half a century. Prof. Hansen pretends, without truth, that when the expressions are broken up and developed according to powers of m, they diverge; and he accounts in this way for the disagreement of his value of the secular inequality from that of Mr. Adams and Delaunay; but Mr. Adams, by calculating the value in both ways, has shown that this argument is fallacious; and M. Delaunay has made this convergence sensible by exhibiting as follows the numerical values of the coefficients of the different powers of m in the secular inequality. See Comptes Rendus, 25 April, 1859.

m^3	m^4	m^5	m^6	m^7	m^8
″	″	″	″	″	″
+10·659	−2·343	−1·582	−·711	−·247	−·062

The terms decrease progressively as the approximation is carried to

higher terms; thus in the coefficient of the *Evection* of which the total value is 4585·″6, the part composed of terms

of the second	order	amounts	to	3173″·4
„ third	„	„		1040 ·4
„ fourth	„	„		295 ·0

and, lastly, the part composed of terms of the eighth order amounts to ·″107. (See Letter of M. Plana to M. Biot, dated the 16th October, 1857, where this objection of M. Hansen is fully treated of.

The importance of maintaining the literal development is well illustrated by what happened recently in regard to the secular equation. Mr. Adams communicated to the Astronomical Society his expression for the secular inequality, in which there occurred the term

$$\frac{17053741}{576} m^7$$

When M. Delaunay saw this, he compared it with the result at which he had arrived independently, and he found that his coefficient differed from that of Mr. Adams by $\frac{50000}{576}$; upon which he examined his calculations, and not finding any error, he wrote to Mr. Adams, to inquire whether he could be quite sure his figures were right. Upon this, Mr. Adams examined his papers, and found that, by inadvertence, he had transcribed a wrong figure in one of the operations, which he had made in calculating the coefficient of m^7; and when this fault was corrected, his result became identical with that of M. Delaunay.

Mr. Main seems to think that I discontinued to work at the Lunar theory, from want of leisure. This is not the case. When I had completed my Part IV., I had planned everything. I had given methods applicable to every kind of periodic inequality, and examples of their application. I had verified all Plana's terms to the fourth order inclusive, and a great many beyond. I had satisfied myself that few, if any, numerical errors of consequence remained to be detected in Plana's coefficients; but there remained most tedious and difficult numerical computations which M. de Pontécoulant was well able to perform; and which I knew, from his letters to me, he would complete.

But this is not all. I wished to be quite sure that the earlier terms were right. If a coefficient is wrong at the commencement, the error infests many of the succeeding terms, and much of the work has to be done over again.

In my opinion, the aim of the theoretical astronomer, even although

his work should be despised by the ignorant or the impertinent, should not be to rush to the construction of Tables and the comparison of predicted places with observation; but to lay the foundations of his work with such solidity, that they may bear with safety all the weight which may be thrown upon them afterwards.

> Quod non imber edax, non Aquilo impotens
> Possit diruere, aut innumerabilis
> Annorum series, et fuga temporum.

It is natural, however, that astronomers, especially those who are principally occupied with observations, should wish to apply their own test, which is the easiest and most obvious, to the results of others; and therefore I urged my friend, M. de Pontécoulant, to publish Tables of the Moon; but M. de Pontécoulant declines having anything to do with the construction of tables, thinking his time can be better employed, and so does M. Plana. We have, indeed, been reproached, on several occasions, with having failed to produce tables founded upon our theory. M. de Pontécoulant says:

"Voilà certes un singulier reproche; et depuis quand a-t-on exigé que les astronomes qui se livrent au travail utile mais fastidieux de réduire en tables les formules de la théorie, travail après tout qui ne demande que de l'attention et de la persévérance, fussent eux-mêmes les auteurs de cette théorie? Ce serait le moyen de n'avoir jamais que des tables en arrière de la science, ou systematiques comme celles de M. Hansen; car ce serait écarter l'eclectisme dans le choix des méthodes, et faire perdre aux plus grands géomètres un temps précieux qui peut être beaucoup plus utilement employé. Lagrange, Laplace, Poisson, ont-ils jamais songéà construire des tables astronomiques, et sans leurs savants travaux, cependant, aurions-nous des tables du Soleil, de Vénus, de Jupiter, de Saturne, etc., aussi parfaites que celles que nous possédons maintenant? Nos connaissances sont trop vastes aujourd'hui pour que l'on puisse exiger d'un seul homme les méditations de la théorie et les labeurs de la pratique: la division du travail, dans le champ de la science comme dans celui de l'industrie est le seul moyen d'arriver, par la réunion de tous les efforts, à des

résultats vraiment grands, vraiment durables, vraiment utiles à l'humanité tout entière." (*Observations sur le Perfectionnement des Tables de la Lune*, p. 19).

The efforts of M. Plana, of Damoiseau, of M. de Pontécoulant, of Mr. Adams, of M. Delaunay, and of myself, have been directed to carrying out the principle so distinctly enunciated by Laplace in vol. iii. of the Méc. Cél. p. 179:—" Il seroit utile pour la perfection des théories astronomiques que toutes les tables (de la lune) derivassent du seul principe de la pesanteur universelle, en n'empruntant de l'observation, que les données indispensables." If the great influence of the Astronomer Royal, the medals of the Astronomical Society, and the money of the government, are employed to procure empirical tables of the moon, and to render useless everything which has been accomplished by us and by all the great mathematicians who have preceded us in the right direction, it behoves those who take the same view of the subject as we do, to impress upon astronomers, upon the publics and upon the government, the mischievous tendency of the proceeding of the Astronomer Royal and Prof. Hansen.

Mr. Longstreth's empirical determinations are eleven in number (see p. 25); there are four of the astronomer Royal already referred to, exclusive of the equation of the centre: of these fifteen coefficients, eleven are identical with ours. The Astronomer Royal publicly stated, at a meeting where I was present, his approval of the symbolical solution, and his wish to see it carried out; if such be the case, there are many ways in which he might promote it. The ephemeris of places of the moon calculated by means of the American Tables, and the comparison of such places with the Greenwich Observations, if not exclusively, at any rate simultaneously with those derived from the empirical tables of Prof. Hansen,* cannot fail to produce beneficial results. The improvement of the lunar theory will be retarded if encouragement or assistance is given to those who pursue one

* The American Ephemeris is calculated for the meridian of Greenwich.

method, and none at all to those who pursue the other? It must be recollected, also, that I am not officially connected with astronomy; it is no part of my duty to improve the practice of navigation. For obvious reasons, I am more in want of help than those who are astronomers by profession, and who have the resources of public observatories at their command.

The following are our eleven coefficients, three of which are identical with those derived from observations by the Astronomer Royal, and the rest with Mr. Longstreth's:

Index.	Argument.	Our value deduced from theory.	Value deduced empirically.	
		$''$	$''$	
1	2τ	2370·799	2370·70	Airy.
3	$2\tau-\xi$	4586·999	4587·01	Airy.
5	$\xi_{,}$	− 668·932	− 669·00	Airy.
7	$2\tau+\xi_{,}$	− 25·036	− 25·00	Longstreth.
11	$\xi+\xi_{,}$	− 109·886	− 110·00	Longstreth.
12	$2\tau-\xi-\xi_{,}$	206·919	206·90	Longstreth.
24	$2\tau-2\xi-\xi_{,}$	9·161	9·20	Longstreth.
64	$2\tau+2\eta$	− 5·728	− 5·776	Longstreth.
65	$\xi-2\eta$	39·427	39·391	Longstreth.
67	$2\tau-\xi-2\eta$	·514	·530	Longstreth.
136	$4\tau-2\xi$	31·153	31·200	Longstreth.

Of the three remaining empirical coefficients of Mr. Longstreth, M. de Pontécoulant has recently, by pushing the approximation further, obtained in two cases the following values, viz.,

		$''$	$''$	
133	$4\tau+\xi$	1·7	1·9	Longstreth.
	$4\tau-2\xi-\xi_{,}$	2·1	3·0	Longstreth.

The coefficient of Arg. 27 remains doubtful: it is difficult to admit the correctness of Mr. Longstreth's coefficient; for he has adopted that which Damoiseau obtained at first, and afterwards corrected; and were it not for this coefficient and for the coefficient of the annual equation, the American Tables give the longitude of the moon very nearly the same as tables would do founded upon our coefficients deduced from theory alone. The reason why the coefficient of the annual equation differs, is, that the Americans took the value determined by the Astronomer Royal empirically in 1848, viz. —670″·3; but the Astronomer Royal's recent determination is —669″·00, which is identical with the value assigned to it by M. de Pontécoulant.

As our value of the variation is identical with that obtained by the Astronomer Royal from the observations, it may be interesting to see

how it is made up; and it affords an instructive example of the manner in which the coefficients of the literal quantities converge: the number within brackets indicates the order of the terms from which the corresponding seconds are derived; the example is taken from M. de Pontécoulant, vol. iv. p. 602:

$$1586''\cdot887[2] + 617''\cdot823[3] + 131''\cdot628[4] + 28''\cdot693[5] + 5''\cdot540[6] + ''\cdot207[7] + ''\cdot021[8] = 2370''\cdot799.$$

This example and many others which could be given, prove the advantages of the literal system.

I think the aged astronomer of Turin can hardly be pleased to see his opinions ignored by the astronomers of England, with the Astronomer Royal and the President of the Astronomical Society at their head, and his great work superseded by empirical tables founded on a numerical solution of the problem. I have his authority to state, that his opinions on this subject remain unchanged; that he considers, as I do, a numerical solution worthless in the present state of astronomy; and that, so far from Professor Hansen's work having advanced the Lunar Theory, it is a step in a retrograde direction. I trust M. Plana will not be deterred, by a love of tranquillity, from again expressing his opinions; and that he will shout, with the warrior of old,—

ὤρινάς μοι θυμὸν ἐνὶ στήθεσσι φίλοισιν,
εἰπὼν οὐ κατὰ κόσμον· ἐγὼ δ' οὐ νῆϊς ἀέθλων,
ὡς σύ γε μυθεῖαι, ἀλλ' ἐν πρώτοισιν ὀΐω
ἔμμεναι, ὄφρ' ἥβῃ τε πεποίθεα, χερσί τ' ἐμῇσι·
νῦν δ' ἔχομαι κακότητι καὶ ἄλγεσι· πολλὰ γὰρ ἔτλην,
ἀλλὰ καὶ ὧς, κακὰ πολλὰ παθὼν, πειρήσομ' ἀέθλων.

In order that the reader may bear in mind the history of the case, I will recapitulate the facts; any impartial person may then appreciate the confidence to which tables founded upon the coefficients of M. de Pontécoulant are entitled.

M. Plana, one of the first astronomers in Europe, publishes coefficients which admit of verification in detail.

Sir John Lubbock verifies a considerable number of the terms in those coefficients, including all the most sensible.

M. de Pontécoulant, coming after Sir John Lubbock, verifies all over again; and where Sir John Lubbock had not succeeded in finding the same figures as M. Plana, went over the calculation again and again, until in many cases he succeeded in finding out an error in Sir John Lubbock's figures, and established the accuracy of Plana's terms disputed by Sir John Lubbock. These corrections were furnished by

M. de Pontécoulant, and were published by Sir John Lubbock as errata.

M. de Pontécoulant continued his investigations; and Sir John Lubbock left to him the honour of bringing this work to a close, one of the most important and arduous in the history of astronomy.

It will be recollected, that M. de Pontécoulant had the advantage of coming after Plana and after me; and as in those cases where he failed in the first instance of arriving at Plana's figures, he bestowed more than ordinary care and attention, I think it may fairly be assumed that his figures, when they differ, are entitled to more confidence than Plana's; the very close agreement between the two, however, shows, I think, invincibly, that if M. de Pontécoulant's coefficients may be rather better, neither can differ much from the truth. With the exception of the annual equation, neither differ two sexagesimal seconds from Burckhardt.

Mr. Airy received the following explanations from M. Plana (see the *Reduction of Greenwich Lunar Observations*, vol. i. p. lxxx.) :—

"The coefficient of $\cos(2\tau + 3\xi)$ appears correct.*

"The coefficient of $\cos \xi$, in the expression for the equatorial parallax, should be $3423''{\cdot}3032$; and that of $\cos 2\xi$, in the terms of the sixth order, $-{\cdot}0543$.

"The coefficient of $\sin(2\tau + \eta)$, in the expression for the latitude, should be $117''{\cdot}580$, instead of $101''{\cdot}708$.

"In the coefficient of $\sin(4\tau - \eta)$, in the expression for the latitude, M. Plana omitted the principal term multiplied by $m^3\gamma$, the coefficient should be $3''{\cdot}402$ instead of $1''.402$. (Plana, vol. i. p. 722.) See Part III. p. 335."

I shall now give some details with respect to the coefficient of the annual equation; and with respect to those coefficients of M. de Pontécoulant which differ from those of M. Plana. When M. de Pontécoulant's coefficients agree with that of M. Plana, their value may be considered as established, unless the development requires to be carried further. I should have wished to recalculate all the terms where a difference occurs between the results of M. Plana and those of M. de Pontécoulant, so as to place clearly in relief the sources of such discrepancies; but my career is drawing to a close; or, in the words of the poet,

> "I near my labours' end,
> Strike sail, and hastening to the harbour tend."

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* This coefficient of M. Plana is, notwithstanding, certainly in error.

The constants employed are as follows:—

Plana, vol. i. pp. 483 and 585.

$m =$	$\cdot 07480130$	log.	$= 8\cdot 8739091$
$e =$	$\cdot 05484721$	„	$= 8\cdot 7391576$
$e_{,} =$	$\cdot 01681013$	„	$= 8\cdot 2256710$
$\gamma =$	$\cdot 09005900$	„	$= 8\cdot 9545271$
$\dfrac{a}{a_{,}} =$	$\cdot 00252064$	„	$= 7\cdot 4015111$

Pontécoulant, vol. iv. p. 589.

$m =$	$\cdot 07480130$	log.	$= 8\cdot 8739091$
$e =$	$\cdot 0547307375$	„	$= 8\cdot 7382312$
$e_{,} =$	$\cdot 0167918226$	„	$= 8\cdot 2250980$
$\gamma =$	$\cdot 089673362$	„	$= 8\cdot 9526635$
$\dfrac{a}{a_{,}} =$	$\cdot 00252551$	„	$= 7\cdot 4023491$

The following are the coefficients according to M. de Pontécoulant.

The figures which have an L. over them were calculated by me previously to the publication of M. de Pontécoulant's work, who recalculated them; their exactitude, therefore, is beyond all doubt. Those which have an A. over them have been verified by Mr. Adams.

But in order to institute an accurate comparison between the results of M. Plana and M. de Pontécoulant, we must first ascertain whether they use the letters e and γ in exactly the same acceptation, by comparing the coefficients of $\cos\xi$ in the reciprocal of the radius vector, of $\sin\xi$ in the longitude, and of $\sin\eta$ in the latitude.

The first comparison gives, neglecting higher terms (see M. de Pontécoulant, vol. iv. p. 568, and M. Plana, vol. i. p. 664),

Pont.

$$e\left\{1+\frac{m^2}{6}-\frac{645}{128}m^3-\frac{152129}{4608}m^4-\left\{\frac{1}{8}+\frac{797}{96}m^2\right\}e^2-\frac{43}{8}m^2e'^2-\frac{83}{128}m^2\gamma^2+\frac{e^4}{192}\right\}$$

Pl.

$$=e\left\{1+\frac{m^2}{6}-\frac{645}{128}m^3-\frac{152129}{4608}m^4-\left\{\frac{1}{8}+\frac{1067}{96}m^2\right\}e^2-\frac{\gamma^2}{4}+\frac{13}{32}e^2\gamma^2-\frac{45}{8}m^2e'^2\right.$$
$$\left.-\frac{265}{384}m^2\gamma^2-\frac{47}{192}e^4-\frac{\gamma^4}{32}\right\}$$

$$e \text{ of Pontécoulant} = e \text{ of Plana}\left\{1-\frac{\gamma^2}{4}-\frac{23}{96}e^4+\frac{3}{8}e^2\gamma^2-\frac{\gamma^4}{32}-\frac{45}{16}m^2e^2-\frac{m^2e_{,}^2}{4}\right\}$$

The second comparison gives, neglecting higher terms (see M. de Pontécoulant, vol. iv. p. 572, and M. Plana, vol. i. p. 574),

Pont.

$$e\left\{\ 2+\frac{3}{2}m^2-\frac{75}{64}m^3-\frac{6659}{256}m^4-\left\{\frac{1}{4}+17m^2\right\}e^2-9m^2e'^2-\frac{51}{32}m^2\gamma^2+\frac{5}{96}e^4\ \right\}$$

Pl.

$$=e\left\{\ 2+\frac{3}{2}m^2-\frac{75}{64}m^3-\frac{6659}{256}m^4-\left\{\frac{1}{4}+17m^2\right\}e^2-\frac{45}{4}m^2e'^2-\frac{\gamma^2}{2}-\frac{63}{32}m^2\gamma^2+\frac{5}{96}e^4 +\frac{23}{16}e^2\gamma^2-\frac{3}{8}\gamma^4\right\}$$

Pont. Pl.

$$e=e\ \left\{\ 1-\frac{\gamma^2}{4}+\frac{11}{16}e^2\gamma^2-\frac{3}{16}\gamma^4-\frac{9}{8}m^2e'^2\right\}$$ a result disagreeing with the former.

The term in m^4 has the coefficient $-\frac{26659}{256}$ in M. de Pontécoulant's work, p. 572, owing to a misprint.

The third comparison gives, neglecting higher terms (see M. de Pontécoulant, vol. iv. p. 583, and M. Plana, vol. i. p. 704),

Pont.

$$\gamma\left\{\ 1+\frac{33}{128}m^3+\frac{241}{512}m^4-\left\{\ 1+\frac{31}{512}m^3\right\}e^2+\frac{27}{8}m^2e'^2+\left\{\frac{1}{8}-\frac{55}{256}m^3\right\}\gamma^2-\frac{e^2\gamma^2}{8} +\frac{3}{64}\gamma^4\right\}$$

Pl.

$$=\gamma\left\{\ 1+\frac{33}{128}m^3+\frac{241}{512}m^4-\left\{\ 1+\frac{31}{512}m^3\right\}e^2+\frac{27}{8}m^2e'^2-\left\{\frac{3}{8}-\frac{5}{128}m^3\right\}\gamma^2+\frac{7}{32}e^2\gamma^2 +\frac{1}{4}\gamma^4\right\}$$

Pont. Pl.

$$\gamma=\gamma\ \left\{\ 1-\frac{\gamma^2}{2}+\frac{17}{32}e^2\gamma^2+\frac{17}{64}\gamma^4+\frac{65}{256}m^2\gamma^2\ \right\}$$

As the letters e and γ have not the same meaning in the works of M. Plana and M. de Pontécoulant, the literal expressions for the coefficients vary even when both are correct in quantities of the fourth and higher orders. In consequence of this circumstance, for example, the coefficients of the terms multiplied by γ^2, Arg. 3, 4, etc., are not the same in the expressions for the longitude given by Plana and Pontécoulant; thus the coefficient of $e\gamma^2$ in the evection is $\frac{39}{16}m$ in the expression of Plana, and $-\frac{3}{2}m$ in that of Pontécoulant, although both are correct.

Neglecting quantities of the fifth order, which only affect the fifth place of decimals,

$$e \text{ of Pontécoulant} = \left\{1 - \frac{\gamma^2}{4}\right\} e \text{ of Plana},$$

which is not the case.

As the Astronomer Royal's recent determination of the equation of the centre is 22639″·06, there can be no doubt that M. Plana's value of the equation of the centre is about 2″·5 too great.

Similarly,

$$\gamma \text{ of Pontécoulant} = \left\{1 - \frac{\gamma^2}{2}\right\} \gamma \text{ of Plana}.$$

According to the Astronomer Royal, the inclination of the moon's orbit assumed by Damoiseau is to be diminished by 2″·35 : as this is the case, M. Plana's coefficient, which is 18465″·5, must be diminished by 2″·45, which reduces it to 18463″·088; and M. de Pontécoulant's coefficient must be increased by 1″·388. The discrepancies to which I have alluded above, weaken materially the means of verification which the literal solution affords. They also destroy the identity which would otherwise be manifest after the literal quantities have been converted into numbers. Thus, for example, in evection we have in Plana,

$$3173''{\cdot}388(2) + 291''{\cdot}514(4);$$

and in Pontécoulant, both being correct,

$$3166''{\cdot}627(2) + 297''{\cdot}506(4).$$

It is only when the discrepancies are much larger than can be accounted for by the difference of the values of the constants, as in some of the following examples, that they arrest attention, and small errors will only be detected by recalculating every term.

As the value of e is obtained by means of the value of the equation of the centre deduced from the observations, and as this already contains, according to Prof. Hansen, the factor 1·0001544, when this is introduced into the coefficient of evection, its value would not be augmented; and if the coefficient of evection be also multiplied by 1·0001544, it would, in fact, contain the square of this quantity implicitly. So that Prof. Hansen must show, not only that the centre of gravity of the moon is distant from its centre of figure, but that the evection is affected thereby in one way, and the equation of the centre in another.

THE LONGITUDE.

There is no error in M. Plana's analytical expression for the longitude in the terms of the first, second, third, or fourth orders.

Arg. 1. 2τ.

Instead of $-\frac{697}{24}m^3e_{,}^2$ (Plana, vol. i. p. 548), the coëfficient should contain the term $-\frac{691}{24}m^3e_{,}^2$ (Lubbock, p. 192); the difference is insensible.

Arg. 3. $2\tau-\xi$.

M. Plana has 1·566(7), which should be 1·838(7); this error being removed, the coefficient becomes 4585″·9.

According to M. de Pontécoulant, the coefficient of $m^4\gamma^2e$ is

$$+\frac{1367059}{18432}.$$

According to M. Plana, the coefficient of $m^4\gamma^2e$ is $-\frac{3469829}{18432}$.

This difference in the formula makes a difference of ″·752 in the numerical coefficient.

Arg. 5. $\xi_{,}$. Pontécoulant, vol. iv. p. 573.

$$+\left\{\begin{array}{l} \left(\overset{\text{L.}}{-3m}+\overset{\text{L.}}{\frac{735}{16}m^3}+\overset{\text{A.}}{\frac{1261}{4}m^4}+\overset{\text{A.}}{\frac{142817}{96}m^5}+\overset{\text{A.}}{\frac{3527665}{576}m^6}\right. \\ \qquad\qquad\qquad\qquad \left.+\frac{964471235}{55296}m^7\right) \\ -e^2\left(\overset{\text{A.}}{\frac{27}{8}m}+\overset{\text{A.}}{\frac{2925}{32}m^2}+\frac{454735}{512}m^3+\frac{39522017}{6144}m^4+\frac{638335067}{16384}m^5\right) \\ +\gamma^2\left(\overset{\text{A.}}{\frac{27}{8}m}-\overset{\text{A.}}{\frac{117}{32}m^2}-\frac{6785}{512}m^3\right)-\frac{27}{8}me_{,}^2 \end{array}\right\}e_{,}\sin\xi_{,} \qquad [5]$$

M. de Pontécoulant calculated with particular care, and by two different methods (Théorie Anal. vol. iv. pp. 177, 188), the first five terms, and found values identical with those of Plana.

M. Plana has, vol. i. p. 575,

$$\frac{964470235}{55296}m^7e_{,}+\frac{6456951283}{294912}m^5e_{,}-\overset{\text{A.}}{\frac{19097}{512}m^3e_{,}\gamma^2}$$

The difference is (Pont.—Pl.), disregarding the second term,

$$\frac{1000}{55296} m^7 e_{,} + \frac{2312}{512} m^3 e_{,} \gamma^2$$

I suspect an error of sign in the term multiplied by $m^5 e_{,}$; but I have not been able to trace it.

The value of this coefficient, according to

Damoiseau, is	673·70
Plana	668·64
Pontécoulant	668·93
Burckhardt	673· 3
Airy	669· 0

The difference between Damoiseau, Plana, and Pontécoulant, arises from some error in Damoiseau, which M. de Pontécoulant has taken great pains to discover, without success. As our value agrees with that determined empirically with so much care by the Astronomer Royal, there can be no doubt of its accuracy.

Arg. 7. $2\tau + \xi_{,}$. Pontécoulant, vol. iv. p. 577.

$$\left\{ -\overset{\text{L.}}{\frac{11}{16}} m^2 - \overset{\text{L.}}{\frac{257}{48}} m^3 - \overset{\text{L.}}{\frac{7337}{576}} m^4 + \overset{\text{A.}}{\frac{124223}{3456}} m^5 + \frac{178759285}{331776} m^6 + \frac{16839339119}{4976640} m^7 \right.$$

$$-\left(\overset{\text{L.}}{\frac{75}{16}} m + \overset{\text{L.}}{\frac{1113}{62}} m^2 - \frac{59161}{1024} m^3 - \frac{4791425}{4096} m^4 \right) e^2$$

$$+\left(\overset{\text{L.}}{\frac{11}{128}} m^2 - \frac{1921}{384} m^3 \right) e'^2 + \left(\overset{\text{L.}}{\frac{3}{16}} m + \overset{\text{L.}}{\frac{73}{64}} m^2 + \overset{\text{L.}}{\frac{24397}{3072}} m^3 \right) \gamma^2$$

$$\left. + \frac{4785}{256} m e^4 + \frac{75}{128} m e^2 e'^2 + \frac{129}{64} m e^2 \gamma^2 - \frac{3}{128} m e'^2 \gamma^2 - \frac{375}{128} m \frac{a^2}{a'^2} \right\} e_{,}$$

When M. Plana's value of e is introduced, instead of $\frac{129}{64} m e^2 \gamma^2$, we get $\frac{279}{64} m e^2 \gamma^2$.

M. Plana has, vol. i. p. 578 (the other terms given by Plana are identical with the above), allowing for his altered value of e,

$$\left\{ \frac{33403086163}{9953280} m^7 + \frac{63385}{1024} m^3 e^2 + \frac{14201441}{4096} m^4 e^2 + \frac{105}{32} m e^4 + \frac{69}{16} m e^2 \gamma^2 \right\} e'$$

the other terms agreeing with de Pontécoulant. The difference between Pontécoulant and Plana is

$$\left\{ \frac{275592075}{9953280} m^7 - \frac{4224}{1024} m^3 e^2 - \frac{9410016}{4096} m^4 e^2 + \frac{3945}{256} m e^4 + \frac{3}{64} m e^2 \gamma^2 \right\} e'$$

which converted into seconds is

$$+\cdot018(6)-\cdot750(7)+\cdot051(8)-=-\cdot731.$$

M. de Pontécoulant has, vol. iv. p. 602,

$$-13\cdot323(3)-11\cdot008(4)-2\cdot213(5)+\cdot647(6)+\cdot707(7)+\cdot154(8)=-25\cdot036.$$

M. Plana should have, vol. i. p. 620,

$$-13\cdot341(3)-11\cdot035(4)-{}^{*}2\cdot214(5)+\cdot595(6)+1\cdot460(7)-\cdot076(8)=-24\cdot611.$$

The difference is

$$\cdot018(3)+\cdot027(4)-\cdot001(5)+\cdot052(6)-\cdot753(7)+\cdot078(8)=-\cdot425.$$

Arg. 11. $\xi+\xi_{\prime}$. Pontécoulant, vol. iv. p. 573.

$$\left\{-\overset{\text{L.}}{\frac{21}{4}}m-\overset{\text{L.}}{\frac{717}{32}}m^2-\overset{\text{A.}}{\frac{3215}{32}}m^3-\overset{\text{A.}}{\frac{2806183}{6144}}m^4-\frac{19962409}{9216}m^5-\frac{18084760319}{1769472}m^6\right.$$

$$\left.-\overset{\text{L.}}{\frac{51}{32}}me^2-\overset{\text{L.}}{\frac{189}{32}}me'^2+\overset{\text{L.}}{\frac{63}{8}}m\gamma^2\right\}ee'$$

When M. Plana's value of e is introduced, instead of $\frac{63}{8}m\gamma^2$, we get $\frac{147}{16}m\gamma^2$, agreeing with Plana.

M. Plana has, vol. i. p. 576,

$$\left\{-\frac{4173895}{6144}m^4-\frac{18012013679}{1769472}m^6\right\}ee$$

The difference is,

$$\left\{\frac{1367712}{6144}m^4-\frac{72746642}{1769472}m^6\right\}ee'$$

which converted into seconds is

$$1\cdot325(6)-\cdot001(8)=1\cdot324.$$

M. de Pontécoulant has, vol. iv. p. 602,

$$-74\cdot445(3)-23\cdot765(4)-9\cdot875(5)-1\cdot301(6)-\cdot50\text{ ind.}=-109\cdot886.$$

In the above, M. de Pontécoulant united by inadvertence the terms of the orders (5) and (6), (7) and (8), and ·50 ind. appears too large; it should be as follows:

$$-74\cdot445(3)-23\cdot765(4)-7\cdot164(5)-2\cdot711(6)-\cdot962(7)-\cdot339(8)-\cdot170\text{ ind.}$$
$$=-109\cdot556.$$

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* M. Plana has 1·214; the error is in the numerical conversion. This correction is due to M. de Pontécoulant.

M. Plana has, vol. i. p. 619,

$$-74{\cdot}702(3)-23{\cdot}847(4)-7{\cdot}030(5)-4{\cdot}046(6)-{\cdot}965(7)-{\cdot}339(8)-{\cdot}17\text{ ind.} =-111{\cdot}099.$$

The difference is

$${\cdot}257(3)+{\cdot}082(4)-{\cdot}139(5)+1{\cdot}336(6)+{\cdot}003(7)=1{\cdot}539.$$

The difference arises chiefly from the coefficient of $m^4 e e_{,}$, in which M. Plana and M. de Pontécoulant disagree. As Mr. Adams has verified this coefficient of M. de Pontécoulant, it must be correct; instead of $-4''{\cdot}046(6)$ in M. Plana's coefficient, we should have $-2''.720$; and thus we get $-109''{\cdot}773$, a result agreeing with that of M. de Pontécoulant.

Arg. 12. $2\tau-\xi-\xi_{,}$. Pontécoulant, vol. iv. p. 577.

$$\left\{\overset{\text{L.}}{\frac{35}{4}}m+\overset{\text{L.}}{\frac{1801}{32}}m^2+\overset{\text{L.}}{\frac{32429}{128}}m^3+\frac{1507947}{2048}m^4+\frac{1202189}{8192}m^5-\frac{260364329333}{1769472}m^6\right.$$

$$\left.-\overset{\text{L.}}{\frac{315}{32}}m\,e^2-\overset{\text{L.}}{\frac{615}{32}}m\,e'^2-\overset{\text{L.}}{\frac{7}{2}}m\,\gamma^2\right\}ee'$$

When M. Plana's value of e is introduced, instead of $-\frac{7}{2}m\gamma^2$, we get $-\frac{91}{16}m\gamma^2$, agreeing with M. Plana.

M. Plana has, vol. i. p. 580,

$$\left\{\frac{1830091}{2048}m^4+\frac{1211197}{8192}m^5+\frac{9159943057}{589824}m^6\right\}ee'$$

The difference is

$$\left\{-\frac{315}{32}m\,e^2-\frac{322144}{2048}m^4-\frac{9008}{8192}m^5-\frac{1443396238}{1769472}m^6\right\}ee'$$

which converted into seconds is

$$-{\cdot}422(5)-{\cdot}937(6)-{\cdot}007(7)-{\cdot}027(8)=-{\cdot}452.$$

M. Plana has $\frac{1830091}{2048}m^4 e e_{,}$, p. 556; but the addition is not exact: this correction is due to M. de Pontécoulant.

M. de Pontécoulant has, vol. iv. p. 602,

$$124{\cdot}071(3)+59{\cdot}695(4)+19{\cdot}208(5)+4{\cdot}369(6)+{\cdot}065(7)-{\cdot}489(8)=206{\cdot}919.$$

The value employed by Damoiseau in his tables is $206''\cdot7$.

M. Plana has, vol. i. p. 621,

$$124\cdot501(3)+59\cdot900(4)+19\cdot436(5)+5\cdot321(6)+\cdot066(7)+\cdot518(8)=209\cdot742.$$

The difference is

$$-\cdot430(3)-\cdot205(4)-\cdot228(5)-\cdot952(6)-\cdot001(7)-\cdot029(8)=-2\cdot823.$$

So that the discrepancy, $2''\cdot823$, between the value of the coefficient given by M. Plana and that given by M. de Pontécoulant arises from the following sources:

M. Plana finds the coefficient of $me^3e_{\prime}$ to be zero, which causes the omission of the term $-\frac{315}{32}me^3e_{\prime}$ (see Lubbock, p. 214), which converted into seconds is $''\cdot422$.

M. Plana has

$$\left\{\frac{1672603}{2048}-\frac{10067}{2048}-\frac{471}{64}+\frac{39}{64}+\frac{99}{8}-\frac{59}{8}=\frac{1830091}{2048}\right\}m^4ee_{\prime}$$

but the addition is erroneous; it should be $\frac{1507947}{2048}$, agreeing with M. de Pontécoulant. The difference converted into seconds is $''\cdot937$.

The value of M. Plana's coefficient, after these corrections, becomes $208''\cdot383$, and the discrepancy is reduced to $1''\cdot464$. This is in great measure produced by the difference of sign of the quantity of the 8th order, which is unusually large. The value employed by Damoiseau in his tables is $206''\cdot7$. This is one of Mr. Longstreth's empirical coefficients, who finds exactly the same value as M. de Pontécoulant. It will be remarked, that in the coefficient of M. de Pontécoulant, the coefficients of m, m^2, m^3, m^4, and m^5, are all positive, while that of m^6 is negative; and if this be established, it shows that it is unsafe to trust to induction. I may here remark, that the quantities added by induction by M. de Pontécoulant require a careful revision; in some instances they appear to be too large, and in others too small. The discrepancy between M. Plana's coefficient and that of M. de Pontécoulant is sometimes due to this cause alone. For example, in the parallactic inequality, of which the numerical value is highly interesting, the analytical expressions of M. Plana and M. de Pontécoulant are very nearly the same; and so would be the numerical value of the coefficient, if they added by induction the same quantity.

Arg. 14. $\xi - \xi_{\prime}$. Pontécoulant, vol. iv. p. 573.

$$\left\{ \overset{\text{L.}}{\frac{21}{4}} m + \overset{\text{L.}}{\frac{1233}{32}} m^2 + \overset{\text{L.}}{\frac{15165}{64}} m^3 + \overset{\text{A.}}{\frac{2844993}{2048}} m^4 + \frac{307187071}{36864} m^5 + \frac{91720676239}{1769472} m^6 \right.$$
$$\left. + \overset{\text{L.}}{\frac{51}{32}} m e^2 + \overset{\text{L.}}{\frac{189}{32}} m e'^2 - \overset{\text{L.}}{\frac{63}{8}} m \gamma^2 \right\} e e'$$

When M. Plana's value of e is introduced, instead of $-\frac{63}{8} m \gamma^2$, we get $-\frac{147}{16} m \gamma^2$, agreeing with M. Plana.

M. Plana has, vol. i. p. 576,

$$\left\{ \frac{307373383}{36864} m^5 + \frac{91915773791}{1769472} m^6 \right\} e e'$$

The difference is

$$\left\{ -\frac{186312}{36864} m^5 - \frac{195097552}{1769472} m^6 \right\} e e'$$

which converted into seconds is

$$-\cdot 002(7) - \cdot 037(8) = -\cdot 039.$$

M. de Pontécoulant has, vol. iv. p. 602,

$$74\cdot 445(3) + 40\cdot 868(4) + 26\cdot 238(5) + 5\cdot 420(6) + \cdot 72 \text{ ind.} = 147\cdot 691.$$

In the above, M. de Pontécoulant united by inadvertence the terms of the orders (5) and (6), (7) and (8); it should stand as follows:

$$74\cdot 445(3) + 40\cdot 868(4) + 17\cdot 994(5) + 8\cdot 244(6) + 3\cdot 699(7) + 1\cdot 721(8) + \cdot 72 \text{ ind.}$$
$$= 147\cdot 691.$$

M. Plana has, vol. i. p. 619,

$$74\cdot 702(3) + 41\cdot 009(4) + 17\cdot 897(5) + 8\cdot 286(6) + 3\cdot 714(7) + 1\cdot 731(8) + \cdot 72 \text{ ind.}$$
$$= 148\cdot 059.$$

The difference is

$$-\cdot 257(3) - \cdot 141(4) - \cdot 096(5) - \cdot 042(6) - \cdot 015(7) - \cdot 010(8) = -\cdot 561.$$

Arg. 22. $2\overset{*}{\tau} + 3\xi$. Pontécoulant, vol. iv. p. 578.

$$\left\{ \overset{\text{L.}}{\frac{779}{192}} m^2 + \frac{7351}{576} m^3 \right\} e^3 \text{ (Lubbock, p. 154.)}$$

M. Plana has, vol. i. p. 580,

$$\frac{1093}{64} m^2 e^3$$

The difference is

$$\left\{-\frac{2500}{192}m^2+\frac{7351}{576}m^3\right\}e^3$$

which converted into seconds is

$$-2{\cdot}479(5)-{\cdot}182(6)=-2{\cdot}297.$$

M. de Pontécoulant has, vol. iv. p. 603,

$${\cdot}768(5)+{\cdot}134(6)={\cdot}902.$$

M. Plana has, vol. i. p. 622,

$$3{\cdot}252(5)+{\cdot}057(6)=3{\cdot}309.$$

The difference is

$$-2{\cdot}484(5)+{\cdot}077(6)=-2{\cdot}407.$$

Damoiseau found, at first, 1″·27 for this coefficient; but in his tables he used the corrected coefficient 1″·0, which confirms the result of M. de Pontécoulant. The coefficient of M. Plana is certainly in error. Longstreth has ″·8.

Arg. 24. $2\tau-2\xi-\xi_{,}$. Pontécoulant, vol. iv. p. 578.

$$\left\{\overset{\text{L.}}{\frac{105}{16}}m+\overset{\text{L.}}{\frac{577}{16}}m^2+\frac{359545}{1024}m^3\right\}e^2e'$$

M. Plana has not got the term

$$\frac{359545}{1024}m^3e^2e'$$

which converted into seconds is

$$1''{\cdot}525.$$

M. de Pontécoulant has, vol. iv. p. 603,

$$5{\cdot}093(4)+2{\cdot}093(5)+1{\cdot}525(6)+{\cdot}45\,\text{ind.}=9{\cdot}161.$$

M. Plana has, vol. i. p. 623,

$$5{\cdot}121(4)+2{\cdot}105(5)+{\cdot}536(6)=7{\cdot}762.$$

The difference is

$$-{\cdot}028(4)-{\cdot}012(5)+{\cdot}989(6)+{\cdot}45\,\text{ind.}=1{\cdot}399.$$

Damoiseau found, at first, 8″·99 for this coefficient; but he afterwards used in his tables the coefficient 9″·1, which agrees with that of M. de Pontécoulant.

Arg. 27. $2\tau-2\xi+\xi_{,}$. Pontécoulant, vol. iv. p. 578.

M. de Pontécoulant's coefficient differs more than any other from that employed in the American Tables; and he has therefore recently pushed the approximation further, and found

$$\overset{\text{L.}}{\left\{-\frac{45}{16}m+\frac{193}{32}m^2+\frac{460721}{3072}m^3+\frac{46022593}{19432}m^4\right\}e^3e,}$$

which converted into seconds is

$$-2{\cdot}183(4)+{\cdot}350(5)+{\cdot}651(6)+{\cdot}811(7)=-{\cdot}370.$$

M. Plana, vol. i. p. 623, has only got the first two terms, which are evidently insufficient.

This coefficient, according to Mr. Longstreth, is $+2{\cdot}2$; the difference being $2{\cdot}6$. This is to me unaccountable; for the terms omitted by M. de Pontécoulant are of the 8th order. Mr. Longstreth has taken a value little different from the first value of Damoiseau, which he afterwards reduced to $\cdot 7$ in his tables. In this case the coefficients do not converge: this happens very rarely; if it were otherwise, Prof. Hansen's objection to the literal development would not be without foundation.

Arg. 39. $2\tau-4\xi$. Pontécoulant, vol. iv. p. 578.

$$\frac{35}{8}me^4$$

M. Plana has, vol. i. p. 582,

$$\frac{25}{4}me^4$$

The difference is

$$-\frac{15}{8}me^4$$

which converted into seconds is

$$-{\cdot}''261.$$

M. de Pontécoulant has, vol. iv. p. 603,

$$\cdot 606(5).$$

M. Plana has, vol. i. p. 624,

$$\cdot 873(5).$$

The difference is

$$-{\cdot}267(5).$$

Arg. 64. $2\tau+2\eta$. Pontécoulant, vol. iv. p. 578.

$$\overset{\text{L.}}{\left\{-\frac{11}{32}m^2-\frac{59}{48}m^3-\frac{195}{64}me^2+\frac{3}{64}m\gamma^2\right\}\gamma^2}$$

M. Plana has, vol. i. p. 579,

$$-\frac{35}{64}me^2\gamma^2$$

and he has not got the term

$$-\frac{59}{48}m^3\gamma^2$$

The difference is

$$-\frac{160}{64}m\,e^2\gamma^2-\frac{59}{48.}m^3\gamma^2$$

which converted into seconds is

$$-{\cdot}''941-{\cdot}''861=-1{\cdot}802.$$

M. de Pontécoulant has, vol. iv. p. 603,

$$-3{\cdot}190(4)-1{\cdot}938(5)-{\cdot}6\,\text{ind.}=-5{\cdot}728.$$

M. Plana has, vol. i. p. 621,

$$-3{\cdot}218(4)-{\cdot}158(5)\qquad\qquad=-3{\cdot}376.$$

The difference is

$${\cdot}128(4,-1{\cdot}780(5)-{\cdot}6\,\text{ind.}=-2{\cdot}352.$$

M. de Pontécoulant's coefficient is confirmed by Burckhardt and Damoiseau.

Arg. 65. $\xi-2\eta$. Pontécoulant, vol. iv. p. 574.

$$\left\{\overset{\text{L.}}{\frac{3}{4}}-\overset{\text{L.}}{\frac{135}{32}}m-\frac{69}{256}m^2-\frac{2867}{2048}m^3\right\}e\gamma^2$$

M. Plana has, vol. i. p. 575,

$$\left\{-\frac{1077}{256}m^2-\frac{21387}{2048}m^3+\frac{3}{8}\gamma^2-\frac{405}{128}m\gamma^2-\frac{61}{32}e^2+\frac{2025}{128}m\,e^2\right\}e\gamma^2$$

M. de Pontécoulant has, vol. iv. p. 602,

$$68{\cdot}084(3)-28{\cdot}647(4)-{\cdot}137(5)-{\cdot}053(6)+{\cdot}18\,\text{ind.}=39''{\cdot}427.$$

Damoiseau used $39''{\cdot}4$ in his tables.

M. Plana has, vol. i. p. 615,

$$68{\cdot}817(3)-28{\cdot}968(4)-2{\cdot}408(5)-{\cdot}250(6)=37''{\cdot}191.$$

The difference is accounted for by the difference in the analytical expression.

Arg. 67. $2\tau-\xi-2\eta$.

M. de Pontécoulant, p. 603, has

$$2{\cdot}547(4)-1{\cdot}937(5)-{\cdot}096(6)={\cdot}514.$$

M. Plana, p. 623, has

$$2{\cdot}574(4)-1{\cdot}958(5)-{\cdot}586(6)={\cdot}030.$$

But there is no term of the sixth order in M. Plana's analytical expression, p. 581. Damoiseau, in his paper, makes the coefficient ·27; but in his tables he makes it ·8. Mr. Longstreth adopts the value above of M. de Pontécoulant.

Arg. 68. $2\tau - \xi + 2\eta$.

M. de Pontécoulant, p. 603, has

$$-6{\cdot}366(4) - 2{\cdot}413(5) - {\cdot}442(6) = -9{\cdot}221.$$

M. Plana, p. 623, has

$$-6{\cdot}435(4) - 2{\cdot}439(5) - {\cdot}510(6) = -9{\cdot}384.$$

But there is no term of the sixth order in M. Plana's analytical expression, p. 581.

M. de Pontécoulant has $-\frac{5}{16}m$, p. 578; M. Plana has, p. 581 $-\frac{15}{16}m$, which I have verified; and in the numerical reduction, M. de Pontécoulant has evidently used $-\frac{15}{16}m$; so probably $-\frac{5}{16}$ is a misprint.

Arg. 72. $\xi_{,} + 2\eta$.

M. Plana has $\frac{153}{128}m^2 e_{,}\gamma^2$; instead of which, this term should be $\frac{201}{128}m^2 e_{,}\gamma^2$ (Lubbock, p. 268), which converted is ·247(5), instead of ·152(5) (Plana, vol. i. p. 620); and it gives for the coefficient $''{\cdot}755$.

Arg. 133. $4\tau + \xi$. Pontécoulant, vol. iv. p. 580.

M. Plana has not carried the approximation to a sufficient extent.

M. de Pontécoulant has recently, at my request, carried the approximation to a greater extent, and has found

$$\overset{\text{L.}}{\left\{ \frac{309}{128}m^4 + \frac{15403}{960}m^5 + \frac{15227467}{230400}m^6 + \frac{11685}{512}m^3 e^2 \right\}} e$$

which reduced to seconds is

$$''{\cdot}85318(5) + ''{\cdot}4242(6) + ''{\cdot}1306(7) + {\cdot}3230(6) = 1''{\cdot}731,$$

which is close to $1''{\cdot}9$, the value adopted by Mr. Longstreth; and although Damoiseau's first value is $-''{\cdot}43$, he afterwards made it $+1''{\cdot}9$, agreeing with the above.

Mr. Adams has informed me, that he also has obtained the value $1''{\cdot}9$ for this coefficient.

Arg. 101. τ.

M. de Pontécoulant has, vol. iv. p. 603,

$-73{\cdot}061(3)-33{\cdot}883(4)-11{\cdot}012(5)-4{\cdot}370(6)-1{\cdot}839(7)-{\cdot}82$[illegible]$(8)$

$-{\cdot}7\ \text{ind.}\left(1-\frac{1}{38}\right)=-122''{\cdot}378.$

M. Plana has, vol. i. p. 624,

$-72{\cdot}920(3)-33{\cdot}818(4)-10{\cdot}939(5)-4{\cdot}393(6)-1{\cdot}847(7)-{\cdot}829(8)$

$-{\cdot}5\ \text{ind.}\left(1-\frac{1}{44}\right)=-122''{\cdot}110.$

The analytical expressions are almost identical.

M. Plana's value of $\frac{a}{a_{\prime}}$ is ·00252064; that of M. de Pontécoulant is ·00252551, corresponding to the solar parallax, 8·63221 (vol. iv. p. 606; this difference makes a difference of about ·2 in the value of the coefficient. M. de Pontécoulant takes $\frac{1}{75}$ for the ratio of the mass of the moon to the sum of masses of the earth and of the moon (vol. iv. p. 589). M. Plana takes $\frac{1}{88}$ for this quantity. The former adds $+3''{\cdot}211$ to the coefficient; the latter, $2''{\cdot}772$. The observations of the tides which I employed gave me a greater value, probably about $\frac{1}{67{\cdot}3}$ M. de Pontécoulant adds —·7 ind., which appears too much; if, with M. Plana, we make it —·5, then the coefficient of M. de Pontécoulant is $122''{\cdot}178$, agreeing with M. Plana.

If we take 124·7 for the value of the coefficient, which is the Astronomer Royal's last empirical determination,

with $\frac{1}{75}$ this gives for the Solar Parallax $8''{\cdot}8103$;

with $\frac{1}{67{\cdot}3}$ „ „ $8''{\cdot}8406$;

values which appear too large to be admissible.

Arg. 136. $4\tau-2\xi$. Pontécoulant, vol. iv. p. 584.

L.

$$\left\{\frac{1125}{256}m^2+\frac{18495}{512}m^3+\frac{1755883}{8192}m^4+\frac{66847963}{61440}m^5\right\}e^2$$

M. Plana has, vol. i. p. 584,

$$\left\{\frac{29055}{512}m^3+\frac{1080043}{8192}m^4+\frac{73818043}{61440}m^5+\frac{466018749241}{235929600}m^6\right\}e^2$$

The difference is

$$\left\{-\frac{10560}{512}m^3+\frac{675840}{8192}m^4-\frac{6970080}{61440}m^5-\frac{466018749241}{235929600}m^6\right\}e^2$$

which converted into seconds is

$$-5{\cdot}356(5)+1{\cdot}602(6)-{\cdot}165(7)-{\cdot}215(8)=-4{\cdot}134.$$

M. de Pontécoulant has, vol. iv. p. 604,

$$15{\cdot}192(4)+9{\cdot}341(5)+4{\cdot}146(6)+1{\cdot}574(7)+{\cdot}9 \text{ ind.}=31{\cdot}153.$$

M. Plana has, vol. i. p. 626,

$$15{\cdot}254(4)+14{\cdot}737(5)+2{\cdot}567(6)+1{\cdot}746(7)+{\cdot}214(8)=34{\cdot}518.$$

The difference is

$$-{\cdot}062(4)-5{\cdot}396(5)+1{\cdot}579(6)-{\cdot}172(7)+{\cdot}686=-3{\cdot}365.$$

M. Plana's analytical expression appears to be incorrect.

The coefficient $31''{\cdot}153$ of M. de Pontécoulant is rather smaller than Damoiseau's first value, $31''{\cdot}19$. Mr. Longstreth increases this value a little, making it $31''{\cdot}2$. He probably was not aware that Damoiseau, in his tables, makes it $31''{\cdot}0$. It seems probable that Mr. Longstreth did not consult Damoiseau's tables, or was not acquainted with them.

$\text{v}_{,}$ $4\tau-2\xi-\xi_{,}$. Pontécoulant, vol. iv. p. 581.

$$\frac{2625}{128}m^2e^2e_{,}$$

The coefficient employed by Mr. Longstreth is $3''{\cdot}0$; according to M. Plana and M. de Pontécoulant, it is $1''{\cdot}2$.

M. de Pontécoulant, at my request, has recently carried the approximation to a greater extent, and has found

$$\frac{2625}{128}m^2e^2e_{,}+\frac{224115}{1024}m^3e^2e_{,}$$

which reduced to seconds is

$$1''{\cdot}191(5)+''{\cdot}9599=2''{\cdot}151.$$

Mr. Longstreth has adopted $3''$, which is the value of Damoiseau.

I trust that M. Plana will be induced to revise his analytical expressions where they differ from those of M. de Pontécoulant, and give us the benefit of his authority, to settle the values of the coefficients in question. Wherever M. de Pontécoulant or myself differ from M. Plana, we have gone over our work so many times, that we think the error must be on his side.

In p. 24 I have made a remark to the effect, that the Americans

ought to have retained the second place of decimals in the coefficients; but, upon reconsideration, I think it was sufficient to retain only the first.

THE LATITUDE.

There is no error in M. Plana's expression for the latitude in terms of the first, second, or third order; and there are only two errors in terms of the fourth order.

Arg. 147. $2\tau-\eta$. Pontécoulant, vol. iv. p. 611.

In the coefficient of sin $(2\tau-\eta)$, M. Plana has $''\cdot689(5)$, vol. i. p. 719, which should be $2''\cdot753(5)$: this error in the reduction being removed, gives for the coefficient, according to M. Plana, $623''\cdot540$; the difference in the values, which, before I introduced this correction, was $2''\cdot093$, being thereby reduced to $''\cdot029$.

Arg. 148. $2\tau+\eta$.

M. Plana has 20·347(4), which should be 36·219(4); this error in the reduction being removed, gives for the coefficient $117''\cdot580$, agreeing with M. de Pontécoulant. This is the most considerable error which occurs in M. Plana's work, and was pointed out by me, Part III. p. 335.

Arg. 152. $2\tau-\xi+\eta$. Pontécoulant, vol. iv. p. 611.

In the coefficient of sin $(2\tau-\xi+\eta)$, M. Plana has $-\cdot465(6)$, vol. i. p. 719, which should be $2''\cdot637$: this error in the reduction being removed, gives for the coefficient, according to M. Plana, $199''\cdot326$; the difference in the values, which, before I introduced this correction, was $2''\cdot586$, being thereby reduced to $''\cdot516$.

Arg. 159. $2\tau+\xi_{\prime}-\eta$.

The discrepancy arises from differences in the analytical development.

Arg. 163. $2\tau-2\xi-\eta$.

M. Plana has 4·879(5), which should be 3·879(5); this error in the reduction being removed, gives for the coefficient $15''\cdot217$. M. de Pontécoulant has $15''\cdot021$: the difference is owing to ·33 ind. added by M. Plana. This correction is due to M. de Pontécoulant.

Arg. 165. $2\tau+2\xi-\eta$.

Instead of $\frac{555}{256}m^2e^2\gamma$, the coefficient should contain the term $\frac{303}{256}m^2e^2\gamma$ (Lubbock, p. 319); which converted is ·370(5), instead of ·681(5) (Plana, vol. i. p. 720); and gives for the coefficient 2·066, agreeing with M. de Pontécoulant.

Arg. 166. $2\tau+2\xi+\eta$.

M. de Pontécoulant has re-examined his coefficient; and errors in the reduction being removed, he finds it should be 1·426. M. Plana has —·171(6), which should be +·178: this error in the reduction being removed, the coefficient becomes 1″·216; the difference, which, before these corrections, was ·418, being thereby reduced to ·210.

Arg. 169. $2\tau-\xi-\xi_{,}-\eta$.

M. de Pontécoulant has re-examined his coefficient, and finds that it should be 7″·810. There is an error in the reduction, vol. iv. p. 612. M. Plana adds, by induction, ″·2 more than M. de Pontécoulant: this increases the discrepancy.

Arg. 170. $2\tau-\xi-\xi_{,}+\eta$.

The discrepancy, which is ″·351, is in great measure caused by the addition of ″·3 ind. by M. Plana.

Arg. 171. $2\tau+\xi+\xi_{,}-\eta$.

M. Plana has $\left\{\frac{1}{2}m-\frac{11}{4}m^2\right\}ee_{,}\gamma$; which should be

L.

$$\left\{-\frac{3}{8}m-\frac{11}{4}m^2\right\}ee_{,}\gamma \text{ (Lubbock, p. 247):}$$

this being converted gives

$$-\cdot480(4)-\cdot264(5)=-''\cdot744$$

instead of +·905; agreeing with M. de Pontécoulant.

Arg. 175. $2\tau-\xi+\xi_{,}-\eta$.

Instead of $-\frac{15}{64}m^2ee_{,}\gamma$, the coefficient should contain the term $-\frac{3}{4}m^2ee_{,}\gamma$ (Lubbock, p. 310); which converted is —·072(5), instead of —·023(5) (Plana, vol. i. p. 720); and gives for the coefficient —1·994,

Arg. 181. $2\tau-2\xi_{,}-\eta$.

M. Plana has ·313(4), which should be ·940; this error being removed, gives for the coefficient ·940, agreeing with M. de Pontécoulant, to whom this correction is due.

Arg. 183. $2\tau+2\xi_{,}-\eta$.

M. Plana has —·055(4), which should be —·110; this error being removed, gives for the coefficient $-''$·117, agreeing with M. de Pontécoulant, to whom this correction is due.

Arg. 185. $\tau-\eta$.

M. de Pontécoulant has —·021(6), which should be —·406: this error in the reduction being removed, gives for the coefficient —4″·514; and the difference, which was ·489, is reduced to ″·104.

Arg. 186. $\tau+\eta$.

M. Plana has —1·245(5), which should be —1·359; this gives for the coefficient —5·130: this correction is due to M. de Pontécoulant.

Arg. 191. $\tau-\xi_{,}-\eta$.

Instead of $\frac{15}{32}me_{,}\gamma\frac{a}{a_{,}}$, the coefficient should contain the term $\frac{15}{16}me_{,}\gamma\frac{a}{a_{,}}$ (Lubbock, p. 310); which converted is insensible.

Args. 193 and 194. $\tau+\xi_{,}-\eta$, and $\tau+\xi_{,}+\eta$.

In the analytical expression for the coefficients and in the reduction M. Plana has the same error, —·033(5), should in each be $-$·331: this gives for the value ″·653, agreeing with M. de Pontécoulant, to whom this correction is due.

Arg. 197.

Instead of $\frac{75}{16}me^3\gamma$, the coefficient should contain the term $\frac{67}{16}me^3\gamma$ (Lubbock, p. 311); which converted is ″·960, instead of 1″·075, agreeing with M. de Pontécoulant.

Arg. 241. $4\tau-\eta$.

M. Plana has omitted the term multiplied by $m^3\gamma$ (Lubbock, p. 248); this error being removed, the coefficient is 3″·553, instead of 1″·402, agreeing with M. de Pontécoulant.

Arg. 244. $4\tau-\xi+\eta$.

The difference, which is $''\cdot941$, is almost entirely caused by the addition of $''\cdot95$ ind. by M. de Pontécoulant.

THE PARALLAX.

There is no error in M. Plana's expression for the reciprocal of the radius vector of the first, second, or third orders, and only one of the fourth order.

Arg. 0.

There is no term $-\frac{45}{16}m^2e^2$ (Lubbock, p. 157).

Arg. 3. $2\tau-\xi$.

M. Plana has $\cdot0404(6)$; this should be $\cdot3082$: this error being removed, gives for the coefficient $34''\cdot189$; and the difference, which was $''\cdot254$, is reduced to $''\cdot014$.

Arg. 6. $2\tau-\xi_{\prime}$.

M. Plana has $\cdot4528(5)$, which should be $\cdot1686$: this error being removed, gives for the coefficient $1''\cdot880$; and the difference, which was $''\cdot289$, is reduced to $''\cdot004$.

Arg. 7. $2\tau+\xi_{\prime}$.

M. Plana has $-\cdot3462(5)-\cdot0376(6)$, which should be $-\cdot1396(4)$ $-\cdot0276(5)$: these errors being removed, give for the coefficient $-''\cdot328$; and the difference, which was $''\cdot217$, is reduced to zero.

Arg. 65. $\xi-2\eta$.

M. Plana has $\cdot9509(4)$; the sign of this quantity should be $-$: this error being removed, gives for the coefficient $-''\cdot709$; and the difference, which was $1\cdot894$, is reduced to $\cdot009$. The sign of Plana's coefficient is wrong in the table given by M. de Pontécoulant, vol. iv. p. 635.

Arg. 73. $2\tau-\xi_{\prime}-2\eta$.

Instead of $-\frac{7}{8}m^2e_{\prime}\gamma^2$ (Plana, vol. i. p. 670), the coefficient should contain the term $-\frac{21}{8}m^2e_{\prime}\gamma^2$ (Lubbock, p. 264).

RECAPITULATION OF CORRECTIONS.

THE LONGITUDE.

Arg. 3.	4585″·648 Pl.	should be	4585″·920,	4586″·999	Pont.
7.	— 23·611 Pl.	„	— 24·611,	25·036	„
11.	—111·099 Pl.	„	—109·773,	—109·886	„
24.	7·762 Pl.	„	9·297,	9·161	„
72.	·585 Pl.	„	·755,	·636	„
133.	·855 Pl.	„	1·280,	1·277	„

The other terms in the longitude which exhibit a considerable difference, and which remain to be cleared up, are Args. 12, 22, 64, 65, and 136.

Since p. 42 was printed, M. de Pontécoulant informs me that M. Plana has 8″·841(6), Arg. 15, which should be 9″·473, also M. Plana adds 2″·5 by induction which seems too much.

THE LATITUDE.

Arg. 147.	621″·476 Pl.	should be	623″·540,	623″·569	Pont.
148.	101·708 Pl.	„	117·580,	117·512	„
152.	196·224 Pl.	„	199·326,	198·810	„
163.	16·217 Pl.	„	15·217,	15·021	„
165.	2·377 Pl.	„	2·066,	2·159	„
166.	1·285 Pont.	„	1·426,		
	·867 Pl.	„	1·216,		
169.	8·311 Pont.	„	7·810,	8·241	Pl.
171.	·905 Pl.	„	— ·744,	— ·739	Pont.
175.	—1·945 Pl.	„	—1·994,	—1·978	„
181.	·627 Pl.	„	·940,	·933	„
183.	— ·062 Pl.	„	— ·117,	— ·117	„
185.	—4·129 Pont.	„	—4·514,	—4·618	Pl.
186.	—5·016 Pl.	„	—5·130,	—5·064	Pont.
193.	·951 Pl.	„	·653,	·648	„
194.	·951 Pl.	„	·653,	·648	„
197.	1·075 Pl.	„	·960,	·950	„
241.	1·402 Pl.	„	3·553,	3·553	„

THE PARALLAX.

Arg. 3.	$33''.921$ Pl.	should be	$34''.189$,	$34''.175$ Pont.
6.	2·165 Pl.	„	1·880,	1·876 „
7.	— ·545 Pl.	„	— ·328,	— ·328 „
65.	1·194 Pl.	„	— ·709.	— ·700 „

By these corrections, we have succeeded in removing all the larger discrepancies which existed between the coefficients of M. Plana and M. de Pontécoulant, in the expressions for the Latitude and for the Parallax: the discrepancies which remain in the expression for the Longitude, appear to be due to errors in the analytical development of M. Plana.

According to the preamble to the American Tables, the values of the daily motions of the moon's mean longitude, the longitude of the moon's perigee, and the longitude of the moon's node derived from the values obtained by Airy in his Memoir upon the *Corrections of the Elements of the Moon's Orbit*, published in the *Memoirs of the Royal Astronomical Society*, vol. xvii., are

$$47435''.02808897$$
$$401.05783886$$
$$-\ 190.63366070$$

These values give

$$(1-c)nt=(.008454885)nt$$
$$(g-1)nt=(.004018854)nt.$$

M. de Pontécoulant found from theory,

$(1-c)nt=(.008450821)nt$, which gives for the daily motion 400·8648
$(g-1)nt=(.004021678)nt$ „ „ —190·6336

The agreement between the theory of M. de Pontécoulant and observation appears to me to be very remarkable.

I have already alluded to the empirical factors introduced by Prof. Hansen (p. 22), which are sufficient to deprive Prof. Hansen's tables of the character of being derived from the Newtonian law of gravitation, or of being other than of the class which are called *empirical;* but these are not the only empiricisms with which Prof. Hansen's tables are affected. Professor Hansen tells us, that the mean motions of the perigee and node employed in his tables are empirical; that is, they " have been deduced from the observations"; and " there will, therefore, be found in the tables, with reference to these two quantities, an empiricism, similarly as in the case of the two inequalities of long period." See Prof. Hansen's Letter to the Astronomer Royal, in the *Monthly Notices* for November, 1854.

On the Secular Variation of the Moon's Motion.

I shall now endeavour to obtain the relation which exists between the constants a and a, h and a, n and n; extending the equations employed in pp. 65 and 134 to the terms multiplied by $e_{\prime}^{2}$, including the calculation of the much controverted term $\frac{3771}{64}m^4 e_{\prime}^{2}$, discovered by Mr. Adams, to which allusion has already been made.

The relation between the constants h and a is again to be obtained from the equations

$$\frac{r^2 d\lambda}{dt}=h-\int\frac{dR}{d\lambda}dt \quad \text{and}$$

$$\frac{r^2 d\lambda^2+dr^2}{dt^2}-\frac{2\mu}{r}+\frac{\mu}{a}+2\int dR=0$$

$$\frac{d^2 r^2}{2dt^2}-\frac{\mu}{r}+\frac{\mu}{a}+2\int dR+r\frac{dR}{dr}=0$$

It is sufficient for the present purpose to take

$$\frac{a}{r}=1-\frac{2}{\mu}\int dR+\frac{r}{\mu}\frac{dR}{dr}.$$

$$dR=\frac{r dR}{dr}d.\frac{\delta r}{r}+\frac{dR}{d\lambda}d.\delta\lambda$$

It follows, from the development of R given in p. 30,

$$\frac{r dR}{dr}=-\frac{m^2}{2}-\frac{3}{4}m^2 e_{\prime}^{2}-\frac{3}{2}m^2\cos 2\tau+\frac{15}{4}m^2 e_{\prime}^{2}\cos 2\tau-\frac{3}{2}m^2 e_{\prime}\cos\xi$$
$$-\frac{21}{4}m^2 e_{\prime}\cos(2\tau-\xi_{\prime})+\frac{3}{4}m^2 e_{\prime}\cos(2\tau+\xi_{\prime})$$

$$d.\frac{\delta r}{r}=\frac{3m^2}{2}e_{\prime}de_{\prime}+5m^2\cos 2\tau\, e_{\prime}de_{\prime}+\frac{3}{2}m^2\cos\xi_{\prime}de_{\prime}$$
$$-\frac{7}{2}m^2\cos(2\tau-\xi_{\prime})\,de_{\prime}+\frac{m^2}{2}\cos(2\tau+\xi_{\prime})\,de_{\prime}$$

$$\frac{dR}{d\lambda}=\frac{3}{2}m^2\sin 2\tau-\frac{15}{4}m^2 e_{\prime}^{2}\sin 2\tau+\frac{21}{4}m^2 e_{\prime}\sin(2\tau-\xi_{\prime})-\frac{3}{4}m^2 e\sin(2\tau+\xi_{\prime})$$

$$d.\delta\lambda=-\frac{55}{8}m^2\sin 2\tau\, e_{\prime}de_{\prime}+\frac{77}{16}m^2\sin(2\tau-\xi_{\prime})de_{\prime}-\frac{11}{16}m^2\sin(2\tau+\xi_{\prime})\,de$$

$$a\frac{dR}{\mu}=\left\{-\frac{3}{4}-\frac{15}{4}-\frac{9}{8}+\frac{147}{16}+\frac{3}{16}-\frac{165}{32}+\frac{1617}{128}+\frac{33}{128}\right\}e_{\prime}de_{\prime}=\frac{735}{64}m^4 e_{\prime}de$$

$$2a\int\frac{dR}{\mu}=\frac{735}{64}m^4 e^2$$

In order to find the coefficient of $m^4 e_{,}^2$ in δR, it is sufficient to take

$$\frac{\delta r}{r}=\frac{m^2}{2}+\frac{3}{4}m^2e_{,}^2-m^2\cos 2\tau+\frac{5}{2}m^2e_{,}^2\cos 2\tau$$

$$+\frac{3}{2}m^2e_{,}\cos\xi_{,}-\frac{7}{2}m^2e_{,}\cos(2\tau-\xi_{,})+\frac{m^2}{2}e_{,}\cos(2\tau+\xi_{,})$$

$$\delta\lambda=\frac{11}{8}m^2\sin 2\tau-\frac{55}{16}m^2e_{,}^2\sin 2\tau+\frac{77}{16}m^2e_{,}\sin(2\tau-\xi_{,})-\frac{11}{16}m^2e_{,}\sin(2\tau+\xi_{,})$$

$$\frac{a\delta R}{\mu}=\left\{-\frac{3}{8}-\frac{3}{8}-\frac{15}{8}-\frac{15}{8}-\frac{9}{8}+\frac{147}{16}+\frac{3}{16}-\frac{165}{64}-\frac{165}{64}+\frac{1617}{128}+\frac{33}{128}\right\}m^4e_{,}^2$$

$$=\frac{735}{64}m^4e_{,}^2$$

$$\frac{ar\,\mathrm{d}R}{\mu\,\mathrm{d}r}=\frac{2aR}{\mu}=\frac{735}{32}m^4e_{,}^5$$

$m=\frac{n_{,}}{\mathrm{n}}$ $\mathrm{n}^2=n^2(1-2m^2-3m^2e_{,}^2)$ $n_{,}^2=n^2m^2(1-2m^2-3m^2e_{,}^2)$ See p. 66 & p. 133.

aR contains the term $-m^2(1-2m^2-3m^2e_{,}^2)\left(\frac{1}{4}+\frac{3}{8}e_{,}^2\right)$

and hence

$$\frac{a}{r}=1+\left\{\frac{2205}{64}+3\right\}$$

$$=1+\frac{2397}{64}m^4e_{,}^2;$$

or, including the term multiplied by m^2,

$$=1-\frac{m^2}{2}-\frac{3}{4}m^2e_{,}^2+\frac{2397}{64}m^4e_{,}^2$$

$$\frac{1}{r^2}\left\{h^2-2h\int\frac{\mathrm{d}R}{\mathrm{d}\lambda}\,\mathrm{d}t+\left\{\frac{\mathrm{d}R}{\mathrm{d}\lambda}\mathrm{d}t\right\}^2\right\}$$

$$+\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2-\frac{2\mu}{r}+\frac{\mu}{a}+2\int\mathrm{d}R=0$$

$$\frac{\mathrm{d}r}{\mathrm{d}t}=2an\left\{m^2-\frac{5}{2}m^2e_{,}^2\right\}\sin 2\tau+7m^2ane_{,}\sin(2\tau-\xi_{,})-m^2ane_{,}\sin 2\tau+\xi_{,})$$

$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2=\left\{-10+\frac{49}{2}+\frac{1}{2}\right\}m^4a^2n^2e_{,}^2=15m^4a^2n^2e_{,}^2$, in which $\frac{\mu}{a^3}$ may be put for n^2.

$\frac{\mathrm{d}R}{\mathrm{d}\lambda}$ would contain no terms of the kind we are seeking, but for the new terms in δr and $\delta\lambda$ discovered by Mr. Adams, multiplied by $\frac{\mathrm{d}e_{,}}{\mathrm{d}t}$, and given by him in the *Phil. Trans.* for 1853, p. 406, which I will now endeavour to obtain.

Integrating by parts, and supposing $e_{,}$ variable, but $\frac{\mathrm{d}e_{,}}{\mathrm{d}t}$ constant, and neglecting $\left(\frac{\mathrm{d}e_{,}}{\mathrm{d}t}\right)^2$,

$$\int e_{\prime} \sin(2\tau-\xi_{\prime})\,dt = -\frac{e_{\prime}}{2n}\cos(2\tau-\xi_{\prime}) + \frac{de_{\prime}}{2ndt}\int\cos(2\tau-\xi_{\prime})\,dt$$

$$= -\frac{e_{\prime}}{2n}\cos(2\tau-\xi_{\prime}) + \frac{de_{\prime}}{4ndt}\sin(2\tau-\xi_{\prime})$$

$$\int e_{\prime}\cos(2\tau-\xi_{\prime})\,dt = \frac{e_{\prime}}{2n}\sin(2\tau-\xi_{\prime}) - \frac{de_{\prime}}{2ndt}\int\sin(2\tau-\xi_{\prime})\,dt$$

$$= \frac{e_{\prime}}{2n}\sin(2\tau-\xi_{\prime}) + \frac{de_{\prime}}{4ndt}\cos(2\tau-\xi_{\prime})$$

$$\int e_{\prime}^2 \sin 2\tau\, dt = -\frac{e_{\prime}^2}{2n}\cos 2\tau + \frac{e_{\prime}de_{\prime}}{ndt}\int\cos 2\tau\, dt$$

$$= -\frac{e_{\prime}^2}{2n}\cos 2\tau + \frac{e_{\prime}de_{\prime}}{2ndt}\sin 2\tau$$

$$\int e_{\prime}^2 \cos 2\tau\, dt = \frac{e_{\prime}^2}{2n}\sin 2\tau - \frac{e_{\prime}de_{\prime}}{ndt}\int\sin 2\tau\, dt$$

$$= \frac{e_{\prime}^2}{2n}\sin 2\tau + \frac{e_{\prime}de_{\prime}}{2ndt}\cos 2\tau$$

From the development of R in p. 30,

$$\frac{a}{\mu}dR = \frac{3}{2}\left\{1 - \frac{5}{2}e_{\prime}^2\right\} m^2 \sin 2\tau\, dt + \frac{21}{4}e_{\prime}\sin(2\tau-\xi_{\prime})dt - \frac{3}{4}e_{\prime}\sin(2\tau+\xi_{\prime})\,dt$$

Integrating by parts, and retaining only the terms multiplied by $\frac{de_{\prime}}{dt}$,

$$\frac{a}{\mu}\int dR = -\frac{15}{8}e_{\prime}\frac{de_{\prime}}{dt}\sin 2\tau + \frac{21}{16}\frac{de_{\prime}}{dt}\sin(2\tau-\xi_{\prime}) - \frac{3}{16}\frac{de_{\prime}}{dt}\sin(2\tau+\xi_{\prime})$$

$\frac{d\lambda}{dt}$ contains the term $-3m^2e_{\prime}\cos\xi_{\prime}$

Integrating by parts,

λ contains the terms,

$$-3me_{\prime}\sin\xi_{\prime} - 3\frac{de_{\prime}}{dt}\cos\xi_{\prime}$$

It is sufficient to take

$$\delta R = \frac{dR}{d\lambda}\delta\lambda \qquad \delta\lambda = -3\frac{de_{\prime}}{dt}\cos\xi_{\prime} \qquad \delta.\frac{rdR}{dr} = 2\delta R$$

$$\frac{dR}{d\lambda} = \frac{3}{2}m^2\sin 2\tau + \frac{21}{4}m^2e_{\prime}\sin(2\tau-\xi_{\prime}) - \frac{3}{4}m^2e_{\prime}\sin(2\tau+\xi_{\prime})$$

$$\delta R = \left\{-\frac{63}{8}+\frac{9}{8}\right\} e_{\prime}\frac{de_{\prime}}{dt}\sin 2\tau - \frac{9}{4}\frac{de_{\prime}}{dt}\sin(2\tau-\xi_{\prime}) - \frac{9}{4}\frac{de_{\prime}}{dt}\sin(2\tau+\xi_{\prime})$$

$$\delta.\frac{rdR}{dr} = -\frac{54}{4}e_{\prime}\frac{de_{\prime}}{dt}\sin 2\tau - \frac{9}{2}\frac{de_{\prime}}{dt}\sin(2\tau-\xi_{\prime}) - \frac{9}{2}\frac{de_{\prime}}{dt}\sin(2\tau+\xi_{\prime})$$

If, considering only the terms multiplied by $\frac{de_{\prime}}{dt}$, we suppose

$$a\delta\frac{1}{r} = r_1\frac{e_{\prime}de_{\prime}}{dt}\sin 2\tau + r_6\frac{de_{\prime}}{dt}\sin(2\tau-\xi_{\prime}) + r_7\frac{de_{\prime}}{dt}\sin(2\tau+\xi_{\prime})$$

dR contains the term $d\lambda\, \delta \frac{dR}{d\lambda}$,

$$\frac{dR}{d\lambda}=\frac{3}{2}m^2 \sin 2\tau + \frac{21}{4}m^2e_, \sin(2\tau-\xi_,) - \frac{3}{4}m^2e_, \sin(2\tau+\xi_,)$$

$$\frac{d.\frac{dR}{d\lambda}}{d\lambda}= 3m^2 \cos 2\tau + \frac{21}{2}m^2e_, \cos(2\tau-\xi_,)-\frac{3}{2}m^2e_, \cos(2\tau+\xi_,)$$

dR contains terms multiplied by $\frac{de_,}{dt}$; to obtain them, it is sufficient to take $\delta\lambda = -3\frac{de_,}{dt}\cos\xi_,$

$$\delta.\frac{dR}{d\lambda}=-\frac{63}{4}m^2 e_, \frac{de_,}{dt}\cos 2\tau + \frac{9}{4}m^2 e_, \frac{de_,}{dt}\cos 2\tau - \frac{9}{2}m^2\frac{de_,}{dt}\cos(2\tau-\xi_,)$$
$$-\frac{9}{2}m^2\frac{de_,}{dt}\cos(2\tau+\xi)$$

$$\int d\lambda\delta.\frac{dR}{d\lambda}=-\frac{54}{4}m^2 e_,{}^2\frac{de_,}{dt}\sin 2\tau - \frac{9}{2}m^2\frac{de_,}{dt}\sin(2\tau-\xi_,)-\frac{9}{2}m^2\frac{de_,}{dt}\sin(2\tau+\xi_,)$$

By substitution in the equation

$$\frac{d^2 r^3 \delta\frac{1}{r}}{dt^2}-\frac{\mu}{r}+\frac{\mu}{a}+2\int dR+\frac{rdR}{dr}=0,$$

we get

$$-20m^2+3r_1-\frac{15}{4}m^2-\frac{53}{4}m^2-\frac{53}{4}m^2=0$$

$$14m^2+3r_6+\frac{21}{8}m^2-\frac{9}{2}m^2-\frac{9}{2}m^2=0$$

$$-\ 2m^2+3r_7-\frac{8}{3}m^2-\frac{9}{2}m^2-\frac{9}{2}m^2=0$$

These equations give

$$r_1=\frac{203}{12} \qquad r_6=-\frac{61}{24} \qquad r_7=\frac{91}{24}$$

which are the values given by Mr. Adams.

I shall now endeavour to obtain the terms in the longitude, multiplied by $\frac{de_,}{dt}$, which are required for the present purpose, by means of the equation which I have generally used in this work, viz.,

$$\frac{d\lambda}{dt}=\frac{h}{r^2}-\frac{1}{r^2}\int\frac{dR}{d\lambda}dt$$

$2a\delta\frac{1}{r}$ contains the terms,

$$5m^2e_,^2\cos 2\tau+7m^2e_,\cos(2\tau-\xi_,)-m^2e_,\cos(2\tau+\xi_,)$$

therefore $\frac{d\lambda}{dt}$ contains these terms; and integrating, the longitude contains the terms,

$$-\frac{5m^2}{2n}\frac{e_,de_,}{dt}\cos 2\tau+\frac{7}{4}\frac{m^2}{n}\frac{de_,}{dt}\cos(2\tau-\xi_,)-\frac{m^2}{4n}\frac{de_,}{dt}\cos(2\tau+\xi_,) \qquad (1)$$

$2\delta\frac{1}{r}$ contains also the terms,

$$\frac{203}{6}\frac{m^2}{n}\frac{e_,de_,}{dt}\sin 2\tau-\frac{61}{12}\frac{m^2}{n}\frac{de_,}{dt}\sin(2\tau-\xi_,)+\frac{91}{12}\frac{m^2}{n}\frac{de_,}{dt}\sin(2\tau+\xi_,)$$

therefore $\frac{d\lambda}{dt}$ contains these terms; and integrating, the longitude contains the terms,

$$-\frac{203}{12}\frac{m^2}{n}\frac{e_,de_,}{dt}\cos 2\tau+\frac{61}{24}\frac{m^2}{n}\frac{de_,}{dt}\cos(2\tau-\xi_,)-\frac{91}{24}\frac{m^2}{n}\frac{de_,}{dt}\cos(2\tau+\xi_,) \qquad (2)$$

$a\frac{dR}{d\lambda}$ contains the terms,

$$\left\{\frac{3}{2}-\frac{15}{4}e_,^2\right\}m^2\sin 2\tau+\frac{21}{4}m^2e_,\sin(2\tau-\xi_,)-\frac{3}{4}m^2e_,\sin(2\tau+\xi_,)$$

Integrating twice, and changing the sign, the longitude contains the terms,

$$-\frac{15}{8}\frac{m^2}{n}\frac{e_,de_,}{dt}\cos 2\tau+\frac{21}{16}\frac{m^2}{n}\frac{de_,}{dt}\cos(2\tau-\xi_,)-\frac{3}{16}\frac{m^2}{n}\frac{de_,}{dt}\cos(2\tau+\xi_,) \qquad (3)$$

$$\frac{d.\frac{dR}{d\lambda}}{d\lambda}=3m^2\cos 2\tau+\frac{21}{2}m^2e_,\cos(2\tau-\xi_,)-\frac{3}{2}m^2e_,\cos(2\tau+\xi_,)$$

$$\delta.\frac{dR}{d\lambda}=\frac{d.\frac{dR}{d\lambda}}{d\lambda}\delta\lambda=-\frac{3de_,}{ndt}\cos\xi_,\left\{3\cos 2\tau+\frac{21}{2}m^2e_,\cos(2\tau-\xi_,)\right.$$
$$\left.-\frac{3}{2}m^2e_,\cos(2\tau+\xi_,)\right.$$

$$=-\frac{54}{4}\frac{m^2e_,de_,}{n\,dt}\cos 2\tau-\frac{9}{2}\frac{m^2}{n}\frac{de_,}{dt}\cos 2\tau-\xi_,)-\frac{9}{2}\frac{m^2}{n}\frac{de_,}{dt}\cos(2\tau+\xi_,)$$

Integrating twice, and changing the sign, the longitude contains the terms,

$$-\frac{54}{16}\frac{m^2}{n}\frac{e_,de_,}{dt}\cos 2\tau-\frac{9}{8}\frac{m^2}{n}\frac{de_,}{dt}\cos(2\tau-\xi_,)-\frac{9}{8}\frac{m^2}{n}\frac{de_,}{dt}\cos(2\tau+\xi_,) \qquad (4)$$

Adding together the four terms which have been found, the longitude contains the terms,

$$\left\{-\frac{5}{2}-\frac{203}{12}-\frac{15}{8}-\frac{54}{16}\right\}\frac{m^2}{n}\frac{e_,de_,}{dt}\cos 2\tau$$

$$+\left\{\frac{7}{4}+\frac{61}{24}+\frac{21}{16}-\frac{9}{8}\right|\frac{m^2}{n}\frac{de_{,}}{dt}\cos(2\tau-\xi_{,})$$

$$+\left|-\frac{1}{4}-\frac{91}{24}-\frac{3}{16}-\frac{9}{8}\right|\frac{m^2}{n}\frac{de_{,}}{dt}\cos(2\tau+\xi)$$

$$=-\frac{74}{3}\frac{m^2}{n}\frac{e_{,}de_{,}}{dt}\cos 2\tau+\frac{215}{48}\frac{m^2}{n}\frac{de_{,}}{dt}\cos(2\tau-\xi_{,})-\frac{257}{48}\frac{m^2}{n}\frac{de_{,}}{dt}\cos(2\tau+\xi_{,})$$

which are the terms given by Mr. Adams.

$$\delta\frac{dR}{d\lambda}=\frac{r\,d.\frac{dR}{d\lambda}}{dr}\frac{\delta r}{r}+\frac{d.\frac{dR}{d\lambda}}{d\lambda}\delta\lambda$$

$$\frac{d.\frac{dR}{d\lambda}}{dr}=\frac{dR}{d\lambda}\frac{\delta r}{r}=\frac{3}{2}m^2\sin 2\tau-\frac{15}{4}m^2e_{,}^2\sin 2\tau+\frac{21}{4}m^2e_{,}\sin(2\tau-\xi_{,})-\frac{3}{4}m^2e_{,}\sin(2\tau+\xi_{,})$$

$$\frac{\delta r}{r}=-\frac{203}{12}m^2e_{,}\frac{de_{,}}{dt}\sin 2\tau+\frac{61}{24}m^2\frac{de_{,}}{dt}\sin(2\tau-\xi_{,})-\frac{91}{24}m^2\frac{de_{,}}{dt}\sin(2\tau+\xi_{,})$$

$$\frac{d.\frac{dR}{d\lambda}}{d\lambda}=3m^2\cos 2\tau-\frac{15}{2}m^2e_{,}^2\cos 2\tau+\frac{21}{2}m^2e_{,}\cos(2\tau-\xi_{,})-\frac{3}{2}m^2e_{,}\cos(2\tau+\xi_{,})$$

$$\delta\lambda=-\frac{74}{3}m^2e_{,}\frac{de_{,}}{dt}\cos 2\tau+\frac{215}{48}m^2\frac{de_{,}}{dt}\cos(2\tau-\xi_{,})-\frac{257}{48}m^2\frac{de_{,}}{dt}\cos(2\tau+\xi_{,})$$

$$\delta\frac{dR}{d\lambda}=\left\{-\frac{203}{16}+\frac{427}{64}+\frac{91}{64}-\frac{74}{2}+\frac{1505}{64}+\frac{257}{64}\right\}m^4e_{,}\frac{de_{,}}{dt}$$

$$=\frac{900}{64}m^4e_{,}\frac{de_{,}}{dt}$$

Multiplying by dt and integrating, we get

$$-\frac{450}{64}m^4e_{,}^2$$

$$\left\{\int\frac{dR}{d\lambda}dt\right\}^2=\left\{-\frac{45}{32}+\frac{441}{128}+\frac{9}{128}\right\}m^4e_{,}^2=\frac{135}{64}m^4e_{,}^2$$

$$\frac{2}{r^3}\left\{h\int\frac{dR}{d\lambda}dt\right\}=-\frac{900}{64}m^4e_{,}^2-4\left\{\left\{-m^2+\frac{5}{2}m^2e_{,}^2\right\}\cos 2\tau\right.$$

$$\left.-\frac{7}{2}m^2e_{,}\cos(2\tau-\xi_{,})+\frac{m^2}{2}e_{,}\cos(2\tau+\xi_{,})\right\}$$

$$\left\{\left\{-\frac{3}{4}m^2+\frac{15}{8}m^2e_{,}^2\right\}\cos 2\tau-\frac{21}{8}m^2e_{,}\cos(2\tau-\xi_{,})+\frac{3}{8}m^2e_{,}\cos 2\tau+\xi_{,})\right\}$$

$$=-\frac{900}{64}m^4e_{,}^2-4\left\{-\frac{15}{16}-\frac{15}{16}+\frac{147}{32}+\frac{3}{32}\right\}m^4e_{,}^2=-\frac{585}{32}m^4e_{,}^2$$

$$\frac{2}{r^3}\left\{h\int\frac{dR}{d\lambda}dt-\frac{1}{2}\left\{\int\frac{dR}{d\lambda}dt\right\}^2\right\}=\left\{-\frac{585}{32}-\frac{135}{64}\right\}m^4e_{,}^2=-\frac{1305}{64}m^4e_{,}^2$$

$$a\delta\frac{1}{r}=-\frac{m^2}{2}\ \frac{8}{4}m^3e^2+\left\{m^2\ \ \frac{5}{2}m^2e_,^2\right\}\cos 2\tau\ \ \ \frac{3}{2}m^2e_,\cos\xi$$

$$+\frac{7}{2}m^2e_,\cos(2\tau-\xi_,)-\frac{m^2}{2}e\ \cos(2\tau+\xi_,)$$

$$a^2\left(\delta\frac{1}{r}\right)^2=\left\{\frac{3}{4}-\frac{5}{2}+\frac{9}{8}+\frac{49}{8}+\frac{1}{8}\right\}m^4e_,^2=\frac{45}{8}m^4e_,^2$$

$$\int\frac{dR}{d\lambda}dt=\left\{-\frac{3}{4}m^2+\frac{15}{8}m^2e_,^2\right\}\cos 2\tau-\frac{21}{8}m^2e_,\cos(2\tau-\xi_,)+\frac{3}{8}m^2e_,\cos(2\tau+\xi_,)$$

$$\delta.\frac{1}{r}\int\frac{dR}{d\lambda}dt=\left\{\frac{15}{16}+\frac{15}{16}-\frac{147}{32}-\frac{3}{32}\right\}m^4e_,^2=-\frac{45}{16}m^4e_,^2$$

$$h^2\left\{1+2r_0+\frac{45}{8}m^4e_,^2\right\}=a\mu\left\{1+2r_0-15m^4e_,^2-\frac{1305}{64}m^4e_,^2-\frac{735}{64}m^4e_,^2\right\}$$

$$=a\mu\left\{1+2r_0-\frac{3000}{64}m^4e_,^2\right\}$$

$$h^2=a\mu\left\{1+2r_0-\frac{3000}{64}m^4e_,^2\right\}\left\{1-\overset{*}{2}r_0+4r_0^2-\frac{45}{8}m^4e_,^2\right.$$

$$=a\mu\left\{1-\left\{\frac{45}{8}+\frac{3000}{64}\right\}m^4e_,^2\right\}$$

$$=a\mu\left\{1-\frac{3360}{64}m^4e_,^2\right\}$$

$$h=\sqrt{a\mu}\left\{1-\frac{1680}{64}m^4e_,^2\right\}$$

$$\frac{d\lambda}{dt}=\sqrt{\frac{\mu}{a^3}}\left\{1-\frac{1680}{64}m^4e_,^2\right\}\left\{1+2r_0+\frac{45}{8}m^4e_,^2+\frac{585}{64}m^4e_,^2\right\}$$

$$=\sqrt{\frac{\mu}{a^3}}\left\{1-\frac{3}{2}m^2e_,^2+\left\{\frac{4794}{64}+\frac{360}{64}+\frac{585}{64}-\frac{1680}{64}\right\}m^4e_,^2\right\}$$

and adding the term $-m^2$, which was obtained in p. 66,

$$n=\sqrt{\frac{\mu}{a^3}}\left\{1-m^2-\frac{3}{2}m^2e_,^2+\frac{4059}{64}m^4e_,^2\right\}$$

In the expression just found, a is absolutely constant, but $e_,$ is variable; consequently, n will vary, and therefore m likewise, which is connected with it by the equation $m=\frac{n_,}{n}$. Taking the variation of the equation for n, and observing that $\frac{\delta m}{m}=-\frac{\delta n}{n}$, we have

$$\delta n=\sqrt{\frac{\mu}{a^3}}\left\{-2m\delta m-\left\{\frac{3}{2}m^2-\frac{4059}{64}m^4\right\}\delta e_,^2\right.$$

* The sign of this quantity in p. 65 should be $-$.

Putting, in this equation, for $\sqrt{\frac{\mu}{a^3}}$, its approximate value $n(1-m^2)$

$$\frac{\delta n}{n}(1-3m^2)=-\left\{\frac{3}{2}m^2-\frac{4059}{64}m^4\right\}\delta e_,^2$$

$$\frac{\delta n}{n}=-\left\{\frac{3}{2}m^2-\frac{3771}{64}m^4\right\}\delta e_,^2$$

Therefore, if N be the initial value of n, and $E_,$ the corresponding value of $e_,$,

$$n=N-\left(\frac{3}{2}m^2-\frac{3771}{64}m^4\right)n\left(e_,^2-E_,^2\right)$$

The first term multiplied by $e_,^2$, is that used by Laplace to account for the acceleration of the moon's motion due to the variability of the earth's eccentricity; the second is the term due to Mr. Adams, and found by him through the equations in which the true longitude is the independent variable, and was published by him in the *Phil. Trans.* for 1853.

If

$$\sqrt{\frac{\mu}{a^3}}\left\{1-m^2-\frac{3}{2}m^2e_,^2+\frac{4059}{64}m^4e_,^2\right\}$$

be called n, and if $\mathrm{n}=\sqrt{\frac{\mu}{\mathrm{a}^3}}$ (see p. 134, Part II.)

$$\mathrm{a}=a\left\{1-m^2-\frac{3}{2}m^2e_,^2+\frac{4059}{64}m^4e_,^2\right\}^{-\frac{2}{3}}$$

$$=a\left\{1+\frac{2}{3}m^2+m^2e_,^2-\frac{3899}{96}m^4e_,^2\right\}$$

$$\frac{\mathrm{a}}{r}=\left\{1-\frac{m^2}{2}-\frac{3}{4}m^2e_,^2+\frac{2397}{64}m^4e_,^2\right\}\left\{1+\frac{2}{3}m^2+m^2e_,^2-\frac{3899}{96}m^4e_,^2\right\}$$

$$=1+\frac{m^2}{6}+\frac{m^2e_,^2}{4}-\frac{799}{192}m^4e_,^2$$

The preceding calculation proceeds upon the supposition that the higher powers of t may be neglected, and that $e_,$ may be properly represented by an expression of the form

$$e_,=E_,+ft$$

No reasoning appears to me to be requisite to prove that if $e_,$ is to be considered as variable at the end of the calculation, it must be treated as such throughout.

On the Development of R.

In the methods employed in this work, much depends upon the accurate development of the disturbing function; I, therefore, endeavoured to obtain this development as free from error as possible, using for the purpose two separate and independent methods. See p. 29. Nevertheless, M. Cayley succeeded in detecting some trifling mistakes, which occur in terms which can have no sensible influence upon the final result, except in case of the sign of the Arg. 8, which dropped out in going through press. The following is extracted from M. Cayley's paper :

"Arg. 1. Lubbock's coefficient (viz. the coefficient in R) is,

$$-\frac{3}{4}\left(1-\frac{5}{2}e^2-\frac{5}{2}e'^2+\frac{23}{16}e^4+\frac{25}{4}e^2e'^2+\frac{13}{16}e'^4+\frac{5}{12}\frac{a^2}{a'^2}\right)\cos^4\frac{1}{2}$$

which, substituting for $\cos^4\frac{1}{2}$ its value $1-\frac{1}{2}\gamma^2+\frac{7}{16}\gamma^4-$ &c., and developing to the fourth order, gives

$$-\frac{3}{4}\left(1-\frac{5}{2}e^2-\frac{5}{2}e'^2+\frac{23}{16}e^4+\frac{25}{4}e^2e'^2+\frac{13}{16}e'^4-\frac{1}{2}\gamma^2+\frac{5}{4}\gamma^2e^2+\frac{5}{4}\gamma^2e'^2+\frac{7}{16}\gamma^4-\frac{5}{12}\frac{a^2}{a'^2}\right)$$

which, in fact, agrees with the value given suprà. I have only referred to this term in order to make the reduction.

Arg. 8. Lubbock's coefficient is,

$$-\frac{1}{8}\left(1-\frac{1}{3}e^2+\frac{3}{2}e'^2-\frac{3}{2}\gamma^2\right)e^2$$

the exterior sign should be + instead of —. The term is given with the correct sign, Pontécoulant, Arg. 2.

Arg. 18. Lubbock's coefficient is,

$$-\frac{51}{8}\left(1-\frac{5}{2}e^2-\frac{115}{51}e'^2-\frac{1}{2}\gamma^2\right)\cos^4\frac{1}{2}$$

which, developed to γ^2, would be

$$-\frac{51}{8}\left(1-\frac{5}{2}e^2-\frac{115}{51}e'^2-\gamma^2\right)$$

I make it

$$-\frac{51}{8}\left(1-\frac{5}{2}e^2-\frac{115}{51}e'^2-\frac{1}{2}\gamma^2\right)$$

The remaining differences are :

No. of Arg. Lubbock.	Lubbock's coefficient.	Coefficient from Development suprà.
58	$+\frac{45}{64}\,e\,e'^3$	$-\frac{845}{64}\,e\,e'^3$
59	$+\frac{591}{64}\,e'^4$	$-\frac{77}{32}\,e'^4$
60	$-\frac{2453}{128}\,e^4$	$-\frac{1599}{64}\,e'^4$
61	$+\frac{741}{128}\,e'^4$	$-\frac{1}{32}\,e^4$
62	$-\frac{3}{8}\left(1-\frac{5}{2}e^2+\frac{3}{2}e'^2\right)\gamma^2$	$-\frac{3}{8}\left(1-\frac{5}{2}e^2+\frac{3}{2}e'^2-\gamma^2\right)\gamma^2$
63	$-\frac{3}{8}\left(1+\frac{3}{2}e^2-\frac{5}{2}e'^2+\frac{1}{8}\gamma^2\right)\gamma^2$	$-\frac{3}{8}\left(1+\frac{3}{2}e^2-\frac{5}{2}e'^2-\gamma^2\right)\gamma^2$
95	$-\frac{27}{16}\gamma^2 e'^2$	$-\frac{27}{32}\gamma^2 e'^2$
96	$-\frac{27}{16}\gamma^2 e'^2$	$-\frac{27}{32}\gamma^2 e'^2$
101	$-\frac{3}{8}\left(1+3e^2+3e'^2-\frac{11}{4}\gamma^2\right)$	$-\frac{3}{8}\left(1+2e^2+2e'^2-\frac{11}{4}\gamma^2\right)$
115	$-\frac{15}{128}\gamma^2$	$-\frac{15}{32}\gamma^2$
123	$-\frac{225}{16}\,e\,e'$	$+\frac{225}{16}\,e\,e'$

The greater part of the discordant terms do not occur in Pontécoulant's development, which is not carried so far; and the only differences which I find in the coefficiens of Pontécoulant's R $(=\Omega)$ are, as regards the arguments 18, 57, 70, corresponding respectively to Lubbock's arguments 62, 63, 101, included in the preceding table, and for which Pontécoulant's coefficients, correcting for the change of sign, correspond with those given by Lubbock."

The following corrections are due to M. Delaunay:

The coefficient of $\frac{a^4}{a_,^5}$, Arg. 5 should be $-\frac{3.45}{4.48}$ instead of $-\frac{3.35}{4.48}$.

Arg. 7 " $-\frac{3.15}{8.32}$ " $-\frac{3.5}{8.4}$.

In developing R in the lunar theory, it is only necessary at first to calculate the coefficient of the terms which are multiplied by *either* e or $e_,$; all the terms which have *both*, may easily be obtained by the following theorem. If we assume

$$\text{coefficient of } \cos(i\tau+n\xi+m\xi_,)=pe^j e_,^k \text{ coefficient of } \cos(2\tau+m\xi_,)$$

when i, n and j are constant, p is constant, whatever m and k may be, except when $i=0$, $m=0$;

when i, m and k are constant, p is constant, whatever n and j may be, except when $i=0$, $n=0$.

The following Table contains examples illustrative of this theorem for $j=1$.

n	p							
					$i=0$			
1	-1	2,0	11,5	14,5	29,17	32,17	53,35	56,35
2	$-\frac{1}{4}$	8,0	23,5	26,5	47,17	50,17		
3	$-\frac{1}{8}$	20,0	41,5	44,5				
					$i=1$			
1	$-\frac{1}{2}$	103,101	109,105	111,104				
-1	$-\frac{5}{2}$	102,101	110,105	108,104				
					$i=2$			
1	1	4,1	13,7	16,6	31,19	34,18	55,37	58,36
-1	-3	3,1	15,7	12,6	33,19	30,18	57,37	54,36
2	1	19,1	25,7	28,6	49,19	52,18		
-2	$\frac{5}{2}$	9,1	27,7	24,6	51,19	48,18		
3	$\frac{5}{24}$	22,1	46,7	46,6				
-3	$-\frac{7}{24}$	21,1	45,7	42,6				
					$i=3$			
1	$\frac{3}{4}$	128,126	124,120	126,119				
-1	5	117,116	125,120	123,119				

The table reads thus, suppose $i=2$, $n=1$, $p=1$, therefore

$$R_4=R_1,\ R_{13}=R_7,\ R_{16}=R_6,\ R_{31}=R_{19},\ R_{34}=R_{18},\ R_{55}=R_{37},\ R_{58}=R_{36}.$$

The first term only is here intended to be taken; the value of p is different for different values of j. In the case of $i=0$, $m=0$, the value of p is double; thus, $R_2=-2R_0$, $R_{11}=-R_5$, $R_{14}=-R_5$, &c.

The coefficient of $e^3e_{,}$ $\cos(2\tau-\xi-\xi_{,})$ employed in the next chapter may similarly be obtained from that of $e^3e_{,}$ in the coefficient of $\cos(2\tau-\xi_{,})$, in which $i=2$, $n=1$, $j=3$.

The coefficient of $e^3 \cos(2\tau-\xi)=-\frac{9\cdot13}{4\cdot24}$

$$,, \qquad \cos(2\tau) \quad = \frac{3\cdot5}{4\cdot2}$$

Therefore $p=-\frac{9\cdot13\cdot4\cdot2}{4\cdot24\cdot3\cdot5}=-\frac{13}{20}$

The coefficient of $e^3e_{,}$ $\cos(2\tau-\xi_{,})=\frac{21\cdot5}{16}$

Therefore the coefficient of $e^3e_{,}$ $\cos(2\tau-\xi-\xi_{,})=-\frac{13\cdot21\cdot5}{20\cdot16}=-\frac{63\cdot91}{7\cdot168}$ which is the coefficient given in p. 31.

This theorem escaped me when I made the development given in p. 30, although I noticed a particular case of it.

$$R_3=-\frac{a\,dR_1}{2\,da}-2\,R_1 \qquad \text{See p. 27.}$$

In this example, $i=2$, $n=1$, $j=1$, $m=0$.

$$\frac{a\,dR_1}{da}=2R_1$$

$$R_3=-3R_1 \qquad \text{hence, } p=-3.$$

But R_{18} is derived in precisely the same manner by this mode of developing R from R_7; and we have similarly,

$$R_{18}=-\frac{a\,dR_7}{2\,da}-2R_7$$

$$=-3R_7 \qquad p=-3 \text{ as before,}$$

and so for any other case.

Calculation of the Terms

$$\left\{-\frac{315}{32}me^2-\frac{615}{32}me_{,}^2\right\}ee_{,}\sin(2\tau-\xi-\xi_{,}) \text{ Arg. 12 in the longitude,}$$

by the Methods employed in this Work.

The following extract from the Table in Part I. shows the arguments which, by their combination with other arguments, by addition and subtraction, produce this argument:

2	8
12 { 24, 6	16 } 12

Arg. 12 arises from the combination of Arg. 2 with Args. 24 and 6, and of Arg. 8 with Arg. 26.

The coefficient in the reciprocal of the radius vector is to be determined by means of the equation,

$$\frac{d^2.\mathrm{r}^3}{2dt^2} - \frac{d^2\mathrm{r}^3\,\delta\frac{1}{r}}{dt^2} + \frac{3d^2\mathrm{r}^4\left(\delta\frac{1}{r}\right)^2}{2dt^2} - \&c.$$

$$-\frac{\mu}{r} + \frac{\mu}{a} + 2\int dR + r\left(\frac{dR}{dr}\right) = 0$$

r being the elliptic value of r.

I denote the coefficient corresponding to the n[th] argument in the expression for $a\delta\frac{1}{r}$ by r_n; and by $\mathfrak{r}_n$, that part of the coefficient of the n[th] argument in the development of the quantity $\mathrm{r}^3\delta\frac{1}{r} - \frac{3}{2}\mathrm{r}^4\left(\delta\frac{1}{r}\right)^2 + \&c.$ which is independent of r_n with a contrary sign.

$$\frac{\mathrm{r}^3}{a^3} = 1 + 3e^2 - 3e\left(1 + \frac{3}{8}e^2\right)\cos\xi + \frac{e^3}{8}\cos 3\xi$$

[0] [2]

Hence I see by the table, that in forming $\mathfrak{r}_{12}$, we must consider the terms in $\delta\frac{1}{r}$, of which the arguments are 6 and $\overline{24}$; and it is sufficient to take:

$$a\delta\frac{1}{r} = \left\{\frac{7}{2}m^2 + \frac{35}{4}me^2 + \frac{799}{16}m^2e^2\right\} e_{,}\cos(2\tau - \xi_{,})$$

[6]

$$-\frac{105}{8}m^2e^2e_{,}\cos(2\tau - 2\xi + \xi)$$

[24]

$\frac{3\mathrm{r}^4}{2a^2}\left(\delta\frac{1}{r}\right)^2$ contains the term $\frac{945}{64}m^2e^2e_{,}\cos(2\tau - \xi - \xi_{,})$ See p. 208.

$$\mathfrak{r}_{12} = \frac{21}{4}m^2 + \frac{105}{8}me^2 + \left\{\frac{2397}{32} + \frac{63}{32} - \frac{315}{16} + \frac{945}{64}\right\} m^2e^2 - \frac{369}{32}m^2e_{,}^2$$

$$= \frac{21}{4}m^2 + \frac{105}{8}me^2 + \frac{4605}{64}m^2e^2 - \frac{369}{32}m^2e_{,}^2$$

Let $r_{12} = \frac{35}{8}m + \frac{1269}{64}m^2 + Ame^2 + Bme^2$

$$\left\{\frac{35}{8}m + \frac{1269}{64}m^2 + Ame^2 + Bme_{,}^2\right\}.\left\{(1+3e^2)(1-6m) - 1\right\}$$

$$(1 - 6m)\left\{\frac{21}{4}m^2 + \frac{105}{8}me^2 + \frac{4605}{64}m^2e^2 - \frac{369}{32}m^2e_{,}^2\right\}$$

$$-4\left\{\frac{63}{8}m^2 - \frac{273}{64}m^2e^2 - \frac{1107}{64}m^2e_{,}^2\right\}$$

The last terms arise immediately from the development of R given in p. 30, which has been carefully verified by M. de Pontécoulant, by M. Cayley, and by M. Delaunay; and the terms in r_{12} which I have assumed, are identical with those of M. Plana, and have been verified by M. de Pontécoulant and myself. It is sufficient to take:

$$\frac{aR}{\mu}=\left\{\frac{63}{8}m^2-\frac{273}{64}m^2e^2-\frac{1107}{64}m^2e_{,}^2\right\}ee_{,}\cos(2\tau-\xi-\xi_{,})\quad \text{See p. 30.}$$

$$-\frac{315}{4}+\frac{3807}{64}-6A=-\frac{315}{4}+\frac{4605}{64}+\frac{272}{16}$$

$$-6B=-\frac{369}{32}+\frac{1107}{16}$$

$$A=-\frac{315}{64}\qquad B=-\frac{615}{64}$$

These results have also been obtained by M. de Pontécoulant; but M. Plana has $A=\frac{35}{8}$.

I shall now endeavour to find the corresponding term in the longitude by means of the equation.

$$\frac{d\lambda}{dt}=\frac{h}{r^2}-\frac{1}{r^2}\int\frac{dR}{d\lambda}dt$$

It is sufficient to take,

$$\frac{d.\delta\lambda}{dr}=\frac{2h}{r}\delta\frac{1}{r}-\frac{1}{r^2}\int\frac{dR}{d\lambda}dt$$

$$\frac{a}{r}=1+e\cos\xi+e^2\cos 2\xi$$

$$\frac{a^2}{r^2}=1+2e\cos\xi\qquad a\int\frac{dR}{d\lambda}dt=-\frac{35}{8}me^2e_{,}\cos(2\tau\quad 2\xi-\xi_{,})$$

$$2a\,\delta\frac{1}{r}=\left\{-\frac{315}{32}me^2-\frac{615}{32}me_{,}^2\right\}ee\ \cos(2\tau-\xi-\xi_{,})+\frac{35}{4}me^2e_{,}\cos(2\tau-\xi)$$

$$+\frac{35}{8}mee_{,}\cos(2\tau-\xi-\xi_{,}).\qquad h=\sqrt{\mu a}\left(1-\frac{e^2}{2}\right)$$

Hence the term required,

$$=\left\{-\frac{315}{32}+\frac{35}{4}-\frac{35}{8}-\frac{35}{8}\right\}me^2-\frac{615}{32}me_{,}^2$$

$$=-\frac{315}{32}me^2-\frac{615}{32}me_{,}^2\qquad \text{See p. 214.}$$

This result has also been obtained by M. de Pontécoulant; but according to M. Plana, the coefficient of me^2 is zero.

This discrepancy is probably owing to an error in the equation $\frac{105}{16}-\frac{35}{16}-\frac{35}{8}=0$, p. 440 of M. Plana's second volume; and as this

result depends upon those given in pp. 344, 347, 370, 376, 380, 397, 408, and 417 of the same volume, to arrive at the source of this decrepancy, and to explain it, would require a volume to be written; whereas in the direct method here given, the final result is arrived at, and all the details given in three pages; and recollecting that it is quite useless to seek for an error in my development of R, upon which the result mainly depends, the whole computation can be verified by any one in a very short time.

On the Comparison of the Results obtained by the Direct and the Indirect Method.

The expressions for the coordinates of the Moon in terms of the mean longitude may be obtained either by the Indirect Method formerly in use and developed by M. Plana, or by the Direct Method, which I was the first to introduce, but the form will not of necessity be identical. The quantity e, commonly called the eccentricity, may be determined in various ways, but most easily by means of the equation of the centre, if e_p represent the quantitiy which M. Plana denotes by e, and e_l the quantity which I have called e, and λ_2 is the coefficient of $\sin\xi$ in the longitude, M. Plana finds

$$\lambda_2 = e_p \left\{ 2 + \frac{3}{2}m^2 - \frac{75}{64}m^3 - \frac{e^2}{2} - \frac{\gamma^2}{2} \right\}$$

$$= e_l \left\{ 2 + \frac{5}{2}m^2 + \frac{285}{32}m^3 - \frac{e^2}{2} \right\} \qquad \text{See p. 196.}$$

$$e_p = e_l \left\{ 1 + \frac{m^2}{2} + \frac{645}{128}m^3 + \frac{\gamma^2}{4} \right\}$$

M. Plana's coefficient of evection contains the terms,

$$e_p \left\{ \frac{15}{4}m + \frac{263}{16}m^2 + \frac{50377}{768}m^3 \right\}$$

$$= e_l \left\{ \frac{15}{4}m + \frac{263}{16}m^2 + \left\{ \frac{50377}{768} + \frac{15}{8} \right\} m^3 \right.$$

$$= e_l \left\{ \frac{15}{4}m + \frac{263}{16}m^3 + \frac{51817}{768}m^3 \right\}$$

which is the value given in p. 196. My coefficient of m^4 is also consistent with that of M. Plana, see p. 339. Again in the reciprocal of the radius vector, M. Plana's coefficient of $\cos 2\tau$ contains the terms

$$\frac{15}{4}m\, e_p^2 + \frac{11923}{256}m^3 e_p^2,$$

if e_l be substituted for e_p, this becomes

$$\frac{15}{4} m e_l^2 + \left\{ \frac{11923}{256} + \frac{15}{4} \right\} m^3 e_l^2 = \frac{15}{4} m e_l^2 + \frac{12883}{256} m^3 e_l^2$$

which is the value I found in p. 123.

Again, in the coefficient of $\sin(2\tau - \xi - \xi_{,})$, Arg. 12, M. Plana's coefficient contains the terms,

$$\left\{ \frac{35}{4} m - \frac{91}{16} m \gamma^2 \right\} e_p e_{,}$$

if e_l be substituted for e_p, this becomes,

$$\left\{ \frac{35}{4} m + \left\{ -\frac{91}{16} + \frac{35}{10} \right\} m\gamma^2 \right\} e_l e_{,} = \left\{ \frac{35}{4} m - \frac{7}{2} m\gamma^2 \right\} e_l e_{,}$$

which is the value I found in p. 267.

The difference in the constants cannot explain the discrepancy which exists between my coefficient of the term multiplied by $me^3 e_{,}$, and that of M. Plana (see p. 42), because the coefficients of e_p^3 and e_l^3 in λ_2 are identical.

$$\gamma_p = \gamma_l \left\{ 1 + \frac{m^2}{6} \right\}$$

M. Plana's coefficient of $\sin(2\tau + \eta)$ contains the term

$$\left\{ \frac{11}{16} m^2 + \frac{7063}{1152} m^4 \right\} \gamma_p$$

if γ_l be substituted for γ_p, this becomes

$$\left\{ \frac{11}{16} m^2 + \left\{ \frac{7063}{1152} + \frac{11}{96} \right\} m^4 \right\} \gamma_l = \left\{ \frac{11}{16} m^2 + \frac{7195}{1152} m^4 \right\} \gamma_l$$

which is the value I found in p. 313.

These examples are intended to shew that when the values of λ_2 obtained by the direct and the indirect method are not identical, one is as good a check upon the other as if they were so. M. de Pontécoulant's expression for e is identical with that of M. Plana as regards the terms multiplied by powers of m, but it differs in the coefficient of γ^2. See p. 37.

On the Determination of the Coefficients in the Expression for the Longitude. Args. 24 and 27.

Argument 24, when e^2 only is taken into account, results from the combinations:

$\{0,24\}$ $\{1,23\}$ $\{2,12\}$ $\{3,11\}$ $\{8,6\}$ and $\{9,5\}$

Argument 27, when e^2 only is taken into account, results from the combinations:

$\{0,27\}$ $\{1,26\}$ $\{2,15\}$ $\{3,14\}$ $\{8,7\}$ and $\{9,5\}$

The characteristic δ is supposed throughout to refer to terms multiplied by $e_{,,}$

$$\frac{h\mathrm{d}.\delta\lambda}{\mathrm{d}t}=\frac{\mathrm{d}(2\mathrm{d}.\mathrm{r}\delta r-\mathrm{dr}\delta r)}{\mathrm{d}t^2}+3\int\mathrm{d}R+2\frac{r\mathrm{d}R}{\mathrm{d}r}-\frac{r\mathrm{d}R}{\mathrm{d}r}\frac{\delta r}{r}+\frac{\mathrm{d}.\delta\lambda}{\mathrm{d}t}\int\frac{\mathrm{d}R}{\mathrm{d}\lambda}\mathrm{d}t$$

$$\mathrm{r}=1-m^2\cos 2\tau+\left(-1-\frac{m^2}{3}\right)e\cos\xi+\left\{-\frac{15}{8}m-\frac{155}{32}m^2\right\}e\cos(2\tau-\xi)$$
$$+\left(-\frac{1}{2}-m^2\right)e^2\cos 2\xi$$

$$\delta r=\left\{-\frac{7}{2}m^2-\frac{157}{8}m^3\right\}e_{,}\cos(2\tau-\xi_{,})+\left\{\frac{m^2}{2}+\frac{91}{24}m^3\right\}e_{,}\cos(2\tau+\xi_{,})$$
$$+\left\{\frac{21}{8}m+\frac{741}{64}m^2\right\}ee_{,}\cos(\xi+\xi_{,})$$
$$+\left\{-\frac{35}{8}m-\frac{1045}{64}m^2-\frac{36919}{768}m^3\right\}ee_{,}\cos(2\tau-\xi-\xi_{,})$$
$$+\left\{-\frac{21}{8}m-\frac{1209}{64}m^2\right\}ee_{,}\cos(\xi-\xi_{,})+\left\{\frac{15}{8}m+\frac{65}{64}m^2-\frac{52813}{768}m^3\right\}ee\cos(2\tau-\xi+\xi_{,})$$
$$+\left\{\frac{21}{8}m+\frac{717}{64}m^2\right\}e^2e_{,}\cos(2\xi+\xi_{,})+\left\{-\frac{21}{8}m-\frac{1233}{64}m^2\right\}e^2e_{,}\cos(2\xi-\xi_{,})$$
$$+\left\{\frac{35}{8}m+\frac{925}{32}m^2\right\}e^2e_{,}\cos(2\tau-2\xi-\xi_{,})+\left\{-\frac{15}{8}m+\frac{45}{32}m^2\right\}e^2e_{,}\cos(2\tau-2\xi+\xi_{,})$$

The last two terms differ from those given by M. de Pontécoulant, vol. iv. p. 510, M. de Pontécoulant having omitted the terms arising from $\left(\delta.\frac{1}{r}\right)^2$ given in p. 144.

The following is the coefficient of $\frac{2\mathrm{d}.\mathrm{r}\delta r-\mathrm{dr}\delta r}{\mathrm{d}t}$, Arg. 24.

$$\left\{\frac{35}{4}m+\frac{925}{16}m^2\right\}\qquad\qquad\left\{3m-\frac{3m^2}{2}\right\}$$
$$m^2\left\{\frac{21}{8}m\right\}\qquad\qquad\left\{3m-\frac{3m^2}{2}+1\right\}$$
$$+\left(1+\frac{m^2}{3}\right)\left\{\frac{35}{8}m+\frac{1045}{64}m^2+\frac{36919}{768}m^3\right\}\quad\left\{3m-\frac{3m^2}{2}-\frac{1}{2}+\frac{3}{8}m^2\right\}$$
$$+\left(\frac{15}{8}m+\frac{155}{32}m^2\right)\left\{-\frac{21}{8}m-\frac{741}{64}m^2\right\}\quad\left\{3m-\frac{3m^2}{2}+\frac{1}{2}-m+\frac{3}{8}m^2\right.$$
$$+\left(\frac{1}{2}+m^2\right)\qquad\left\{\frac{7}{2}m^2+\frac{157}{8}m^3\right\}\qquad\left\{3m-\frac{3m^2}{2}-1+\frac{3}{4}m^2\right\}$$

The following is the coefficient of $\dfrac{2\mathrm{d}.r\delta r - \mathrm{d}r\delta r}{\mathrm{d}t}$, Arg. 27.

$$\left\{-\frac{15}{4}m+\frac{45}{16}m^2\right\} \qquad \left\{m-\frac{3m^2}{2}\right\}$$

$$m^2\left\{-\frac{21}{8}m\right\} \qquad \left\{m-\frac{3m^2}{2}-1\right\}$$

$$+\left(1+\frac{m^2}{3}\right)\left\{-\frac{15}{8}m-\frac{65}{64}m^2+\frac{52813}{768}m^3\right\} \qquad \left\{m-\frac{3m^2}{2}-\frac{1}{2}+\frac{3}{8}m^2\right\}$$

$$+\left(\frac{15}{8}m+\frac{155}{32}m^2\right)\left\{\frac{21}{8}m+\frac{1209}{64}m^2\right\} \qquad \left\{m-\frac{3m^2}{2}+\frac{1}{2}-\frac{1}{2}m+\frac{3}{8}m^2\right\}$$

$$+\left(\frac{1}{2}+m^2\right)\left\{-\frac{1}{2}m^2-\frac{91}{24}m^3\right\} \qquad \left\{m-\frac{3m^2}{2}-1+\frac{3}{4}m^2\right\}$$

$$=\left\{-\frac{35}{16}m+\left\{\frac{105}{8}+\frac{105}{4}-\frac{1045}{128}-\frac{315}{128}-\frac{7}{4}=27\right\}m^2+\left\{-\frac{105}{8}-\frac{105}{16}+\frac{2775}{16}\right.\right.$$

$$+\frac{3135}{64}-\frac{945}{64}+\frac{21}{4}-\frac{21}{8}-\frac{36919}{1536}+\frac{175}{192}-\frac{17625}{1024}+\frac{315}{64}-\frac{157}{16}$$

$$\left.\left.=\frac{446567}{3072}\right\}m^3\right\}e^2e_{,}\cos(2\tau-2\xi-\xi_{,})$$

$$+\left\{\frac{15}{16}m+\left\{-\frac{15}{8}-\frac{15}{4}-\frac{65}{128}+\frac{315}{128}+\frac{1}{4}=-\frac{77}{32}\right\}m^2+\left\{\frac{45}{8}+\frac{45}{16}+\frac{45}{16}-\frac{65}{64}+\frac{315}{64}-\frac{1}{4}\right.\right.$$

$$\left.\left.+\frac{21}{8}-\frac{52813}{1536}-\frac{75}{192}+\frac{24645}{1024}-\frac{315}{64}+\frac{91}{48}=\frac{11669}{3072}\right\}m^3\right\}e^2e_{,}\cos(2\tau-2\xi+\xi_{,})$$

The values of R or $\dfrac{r\mathrm{d}R}{\mathrm{d}r}$, containing all the terms required in the following examples are given in pp. 30, 53, 119, 138 and 165.

$\dfrac{\mathrm{d}R}{\mathrm{d}\lambda}$ is given in pp. 59, 141, 181, and 223.

No uncertainty exists with respect to any of the terms required in $\delta\dfrac{1}{r}$ and $\delta\lambda$. The value of $r\delta\dfrac{1}{r}$ given in p. 157 is complete for the purpose except the following terms. $r\delta\dfrac{1}{r}$ contains the following:

$$+\left\{-\frac{63}{16}m-\frac{2271}{128}m^2\right\}e^2e\cos(2\xi+\xi_{,})+\left\{\frac{63}{16}m+\frac{3579}{128}m^2\right\}e^2e\cos(2\xi-\xi_{,})$$

$$+\left\{-\frac{35}{16}m-\frac{1373}{64}m^2\right\}e^2e_{,}\cos(2\tau-2\xi-\xi_{,})+\left\{\frac{15}{16}m+\frac{19}{64}m^2\right\}e^2e_{,}\cos(2\tau-2\xi+\xi_{,})$$

$$+\frac{1575}{128}m^2e^2e_{,}\cos(4\tau-2\xi-\xi_{,})-\frac{675}{128}m^2e^2e_{,}\cos(4\tau-2\xi+\xi_{,})$$

The discrepancy between the value of δR here given, and that given by M. de Pontécoulant, vol. iv. p. 511, arises principally from his accidental omission of the following terms in the reciprocal of the radius vector and in the longitude.

$$\delta\frac{1}{r}=\frac{525}{32}m^2e^2e_{,}\cos(4\tau-2\xi-\xi_{,})-\frac{225}{32}m^2e^2e_{,}\cos(4\tau-2\xi+\xi_{,})$$

$$\delta\lambda=\frac{2625}{128}m^2e^2e_{,}\sin(4\tau-2\xi-\xi_{,})-\frac{1125}{128}m^2e^2e_{,}\sin(4\tau-2\xi+\xi_{,})$$

The characteristic δ is supposed throughout to refer to $e_{,}$.

$$\delta R=\frac{dR}{dr}\delta r+\frac{dR}{d\lambda}\delta\lambda+\frac{dR}{dr_{,}}\delta r_{,}+\frac{dR}{d\lambda_{,}}\delta\lambda_{,}$$

$$=-2Rr\delta\frac{1}{r}+\frac{dR}{d\lambda}\delta\lambda-3R\frac{\delta r_{,}}{r_{,}}-\frac{dR}{d\lambda}\delta\lambda_{,}$$

$$\frac{\delta r_{,}}{r_{,}}=e_{,}\cos\xi_{,} \qquad \delta\lambda_{,}=2e_{,}\sin\xi_{,}$$

Finally, δR

$$=\Big\{-\frac{105}{16}m^2+\Big\{-\frac{35}{32}-\frac{189}{64}-\frac{35}{16}+\frac{189}{32}-\frac{45}{64}-\frac{45}{32}-\frac{315}{64}+\frac{189}{16}-\frac{45}{8}=-\frac{75}{64}\Big\}m^3$$

$$+\Big\{-\frac{1373}{128}-\frac{6813}{512}+\frac{4725}{512}-\frac{1157}{128}-\frac{4725}{256}+\frac{7101}{256}+\frac{315}{128}-\frac{7}{16}-\frac{45}{16}$$

$$-\frac{11007}{512}-\frac{4725}{128}+\frac{7875}{512}+\frac{6453}{128}-\frac{24609}{1024}+\frac{1751}{256}$$

$$=-\frac{25765}{1024}\Big\}m^4\Big\}e^2e_{,}\cos(2\tau-2\xi-\xi_{,})$$

$$+\Big\{\frac{15}{16}m^2+\Big\{\frac{15}{32}+\frac{189}{64}+\frac{15}{16}-\frac{189}{32}-\frac{45}{64}-\frac{45}{32}+\frac{315}{64}-\frac{189}{16}+\frac{45}{8}=-\frac{315}{64}\Big\}m^3$$

$$+\Big\{\frac{19}{128}+\frac{10737}{512}-\frac{2025}{512}+\frac{81}{128}+\frac{2025}{256}-\frac{10449}{256}-\frac{315}{128}+\frac{1}{16}-\frac{45}{16}$$

$$+\frac{18243}{512}-\frac{3375}{512}+\frac{2025}{128}-\frac{11097}{128}-\frac{24609}{1024}-\frac{1751}{256}$$

$$=-\frac{95261}{1024}\Big\}m^4\Big\}e^2e_{,}\cos(2\tau-2\xi+\xi_{,})$$

M. de Pontécoulant's coefficients are, vol. iv. p. 511,

$$\frac{105}{16}m^2+\frac{75}{64}m^3+\frac{45505}{1024}m^4 \qquad \text{Arg. 24.}$$

$$\frac{15}{16}m^2+\frac{315}{64}m^3+\frac{84161}{1024}m^4 \qquad \text{Arg. 27.}$$

The terms multiplied by m^2 and m^3 agree with mine. In order to introduce my eccentricity, $e^2(1+m^2)$ must be substituted for e^2 in the

expression of M. de Pontécoulant. This with the terms omitted accidentally give for the coefficient of m^4,

$$-\frac{45505}{1024}-\frac{105}{16}+\frac{315}{256}+\frac{4725}{512}+\frac{7875}{512}=-\frac{25765}{1024}$$

$$-\frac{84161}{1024}+\frac{15}{16}-\frac{315}{256}-\frac{2025}{512}-\frac{3375}{512}=-\frac{95261}{1024}$$

$$\frac{dR}{d\lambda}=\frac{3r^2}{2r_{,}^3}\sin(2\lambda-2\lambda_{,})$$

$$\delta\frac{dR}{d\lambda}=-\frac{2dR}{d\lambda}r\delta\frac{1}{r}+\left\{-4R-\frac{r^2}{r_{,}^3}\right\}\delta\lambda-3\frac{dR}{d\lambda}\frac{\delta r_{,}}{r_{,}}+\left\{4R+\frac{r^2}{r'^3}\right\}\delta\lambda_{,}$$

$$4R+\frac{r^2}{r_{,}^3}=0m^3+\left\{-\frac{8203}{128}+\frac{1}{2}+\frac{171}{16}+\frac{443}{32}=-\frac{4999}{128}m^4\right\}e^2\cos(2\tau-2\xi)$$

$$\begin{aligned}\delta\frac{dR}{d\lambda}=\Bigg\{&\left\{\frac{189}{32}-\frac{189}{16}+\frac{315}{32}-\frac{189}{8}+\frac{45}{4}=-\frac{135}{32}\right\}m^3\\&+\left\{\frac{6813}{256}+\frac{4725}{256}-\frac{4725}{128}-\frac{7101}{128}+\frac{45}{8}+\frac{11007}{256}\right.\\&\qquad\left.+\frac{7875}{256}-\frac{4725}{64}-\frac{6453}{64}-\frac{5253}{512}+\frac{4999}{128}=-\frac{58265}{512}m^4\right\}e^2e_{,}\sin(2\tau-2\xi-\xi_{,})\\+\Bigg\{&\left\{-\frac{189}{32}+\frac{189}{16}-\frac{315}{32}+\frac{189}{8}-\frac{45}{4}=\frac{135}{32}\right\}m^3\\&+\left\{-\frac{10737}{256}-\frac{2025}{256}+\frac{2025}{128}+\frac{10449}{128}+\frac{45}{8}-\frac{18243}{256}\right.\\&\qquad\left.-\frac{3375}{256}+\frac{2025}{64}+\frac{11097}{64}-\frac{5253}{512}-\frac{4999}{128}=\frac{63743}{512}m^4\right\}e^2e_{,}\sin(2\tau-2\xi+\xi_{,})\end{aligned}$$

M. de Pontécoulant's coefficients are, vol. iv. p. 511,

$$-\frac{105}{8}m^2+\frac{135}{16}m^3+\frac{90185}{512}m^4 \qquad \text{Arg. 24.}$$

$$\frac{15}{8}m^2-\frac{135}{16}m^3-\frac{75503}{512}m^4 \qquad \text{Arg. 27.}$$

The terms multiplied by m^2 and m^3 agree with mine. In order to introduce my eccentricity, $e^2(1+m^2)$ must be substituted for e^2 in the expression of M. de Pontécoulant. This with the terms omitted accidentally gives for the coefficient of m^4,

$$-\frac{90185}{512}+\frac{105}{8}+\frac{4725}{256}+\frac{7875}{256}=-\frac{58265}{512}$$

$$\frac{75503}{512}-\frac{15}{8}-\frac{2025}{256}-\frac{3375}{256}=\frac{63743}{512}$$

which agree with the values given above.

$$\frac{dR}{d\lambda}=\left\{\frac{105}{8}m^2-\frac{135}{32}m^3-\frac{58265}{128}m^4\right\}e^2e\sin(2\tau-2\xi-\xi_{,})$$

$$+\left\{-\frac{15}{8}m^2+\frac{135}{32}m^3+\frac{63743}{256}m^4\right\}e^2e_{,}\sin(2\tau-2\xi+\xi_{,})$$

Integrating, $\int\frac{dR}{d\lambda}dt$

$$=\left\{\frac{35}{8}m-\frac{5}{8}m^2-\frac{27245}{1536}m^3\right\}e^2e_{,}\cos(2\tau-2\xi-\xi_{,})$$

$$+\left\{-\frac{15}{8}m+\frac{45}{8}m^2+\frac{54563}{512}m^3\right\}e^2e_{,}\cos(2\tau-2\xi+\xi)$$

Supposing (see p. 166 and p. 223),

$$\frac{r_{,}dR_{,}}{dr_{,}}=-3R=\left\{\frac{45}{8}m^2+\frac{135}{32}m^3+\frac{24609}{512}m^4\right\}e^2\cos\ 2\tau-2\xi)$$

$$\frac{dR}{d\lambda_{,}}=-\frac{dR}{d\lambda}=\left\{-\frac{15}{4}m^2+\frac{1751}{256}m^4\right\}e^2\sin(2\tau-2\xi)$$

$$\frac{r_{,}dR}{dr_{,}}\frac{dr_{,}}{r_{,}}+2\frac{dR}{d\lambda_{,}}\cos\xi_{,}mndt$$

$$=\left\{-\frac{105}{16}m^3-\frac{135}{64}m^4-\frac{17605}{1024}m^5\right\}e^2e_{,}\sin(2\tau-2\xi-\xi)$$

$$+\left\{-\frac{15}{16}m^3+\frac{135}{64}m^4+\frac{31613}{1024}m^5\right\}e^2e_{,}\sin(2\tau-2\xi+\xi_{,})$$

$$\int\left\{\frac{r_{,}dR}{dr_{,}}dr_{,}+\frac{dR}{d\lambda_{,}}\cos\xi_{,}mndt\right\}$$

$$=\left\{-\frac{35}{16}m^2-\frac{115}{64}m^3-\frac{51865}{3072}m^4\right\}e^2e_{,}\cos(2\tau-2\xi-\xi_{,})$$

$$+\left\{-\frac{15}{16}m^2+\frac{45}{64}m^3+\frac{19193}{1024}m^4\right\}e^2e_{,}\cos(2\tau-2\xi+\xi_{,})\cdot$$

This agrees with the expression of M. de Pontécoulant.

$$\int dR=R+\int\frac{dR}{d\lambda}mndt-\int\left\{\frac{r_{,}dR}{dr_{,}}dr_{,}+\frac{dR}{d\lambda_{,}}\cos\xi_{,}mndt\right\}$$

$$=\left\{\left\{-\frac{105}{16}+\frac{35}{8}+\frac{35}{16}=0\right\}m^2+\left\{-\frac{75}{64}-\frac{5}{8}+\frac{115}{64}=0\right\}m^3\right.$$

$$\left.=\left\{-\frac{25765}{1024}-\frac{27245}{1536}+\frac{51865}{3072}=-\frac{39960}{1536}\right\}m^4\right\}e^2e_{,}\cos(2\tau-2\xi-\xi_{,})$$

$$+\left\{\left\{\frac{15}{16}-\frac{15}{8}+\frac{15}{16}=0\right\}m^2+\left\{-\frac{315}{64}+\frac{45}{8}-\frac{45}{64}=0\right\}m^3\right.$$

$$\left.+\left\{-\frac{95261}{1024}+\frac{54563}{512}-\frac{19193}{1024}=\frac{2664}{512}\right\}m^4\right\}e^2e_{,}\cos(2\tau-2\xi+\xi_{,})$$

The coefficients of m^2 and m^3 are zero.

$$-r\frac{dR}{dr}\frac{\delta r}{r}=2R\,r\delta\frac{1}{r}$$

This quantity is part of δR, which has already been calculated with a contrary sign, and it equals

$$\left\{\frac{21}{64}m^3+\frac{3930}{256}m^4\right\}e^2e_,\cos(2\tau-2\xi-\xi_,)$$
$$+\left\{\frac{99}{64}m^3+\frac{2601}{128}m^4\right\}e^2e_,\cos(2\tau-2\xi+\xi_,)$$

M. de Pontécoulant has

$$-\frac{21}{64}m^3-\frac{13215}{512}m^4 \qquad \text{Arg. 24.}$$

$$-\frac{99}{64}m^3-\frac{7749}{512}m^4 \qquad \text{Arg. 27.}$$

$$\frac{13215}{512}-\frac{315}{256}+\frac{4725}{512}=\frac{3930}{256}$$

$$\frac{7749}{512}+\frac{315}{256}+\frac{2025}{512}=\frac{2601}{128}$$

$$\int\frac{dR}{d\lambda}dt=\left\{-\frac{3}{4}m^2-\frac{3}{4}m^3\right\}\cos 2\tau+\frac{135}{16}m^3e\cos\xi+\left\{\frac{9}{2}m^2+9m^3\right\}\cos(2\tau-\xi)$$
$$+\frac{15}{8}m\,e^2\cos(2\tau-2\xi)$$

$$\frac{d.\delta\lambda}{dt}=-3m^2e_,\cos\xi_,+\left\{-\frac{21}{4}m-\frac{885}{32}m^2\right\}e\,e_,\cos(\xi+\xi_,)+\frac{35}{4}mee_,\cos(2\tau-\xi-\xi_,)$$
$$+\left\{\frac{21}{4}m+\frac{1065}{32}m^2\right\}e\,e_,\cos(\xi-\xi_,)-\frac{15}{4}mee_,\cos(2\tau-\xi+\xi_,)$$
$$+\left\{-\frac{105}{8}m-\frac{4089}{64}m^2\right\}e^2e_,\cos(2\xi+\xi_,)+\left\{\frac{105}{8}m+\frac{5661}{64}m^2\right\}e^2e_,\cos(2\xi-\xi_,)$$
$$+\frac{2625}{64}m^2e^2e_,\cos(4\tau-2\xi-\xi_,)-\frac{1125}{64}m^2e^2e_,\cos(4\tau-2\xi+\xi_,)$$

$$\frac{d.\delta\lambda}{dt}\int\frac{dR}{d\lambda}dt=$$

$$\left\{\left\{\frac{315}{64}-\frac{189}{16}-\frac{45}{16}=-\frac{621}{64}\right\}m^3+\left\{\frac{12267}{512}-\frac{7875}{512}+\frac{315}{64}+\frac{4725}{128}-\frac{7965}{128}-\frac{189}{8}\right.\right.$$
$$\left.\left.-\frac{135}{64}=-\frac{19224}{512}m^4\right\}\right\}e^2e_,\cos(2\tau-2\xi-\xi_,)$$

$$+\left\{\left\{\frac{315}{64}+\frac{189}{16}-\frac{45}{16}=\frac{261}{64}\right\}m^3+\left\{-\frac{16983}{512}+\frac{3375}{512}-\frac{315}{64}-\frac{2025}{128}+\frac{9585}{128}\right.\right.$$
$$\left.\left.-\frac{189}{8}-\frac{135}{64}=\frac{25128}{512}m^4\right\}\right\}e^2e_,\cos(2\tau-2\xi+\xi_,)$$

M. de Pontécoulant has

$$\frac{621}{64}m^3+\frac{11349}{512}m^4 \qquad \text{Arg. 24.}$$

$$-\frac{261}{54}m^3-\frac{21753}{512}m^4 \qquad \text{Arg. 27.}$$

$$-\frac{11349}{512}-\frac{7875}{512}=-\frac{19224}{512} \qquad \frac{21753}{512}+\frac{3375}{512}=\frac{25128}{512}$$

Hence, finally,

$$3\int dR+2\frac{rdR}{dr}-\frac{rdR}{dr}\frac{\delta r}{r}+\frac{d.\delta\lambda}{dt}\int\frac{dR}{d\lambda}dt$$

$$=\left\{-\frac{105}{4}m^2+\left\{-\frac{75}{16}+\frac{21}{64}-\frac{621}{64}=-\frac{225}{16}\right\}m^3\right.$$

$$\left.+\left\{-\frac{39660}{512}-\frac{25765}{256}+\frac{3930}{256}-\frac{19224}{512}=-\frac{51427}{256}\right\}m^4\right\}e^2e\cos(2\tau-2\xi-\xi)$$

$$+\left\{\frac{15}{4}m^2+\left\{-\frac{315}{16}+\frac{99}{64}+\frac{261}{64}=-\frac{225}{16}\right\}m^3\right.$$

$$\left.+\left\{-\frac{7992}{512}-\frac{95261}{256}+\frac{2601}{128}+\frac{25128}{512}=-\frac{81491}{256}\right\}m^4\right\}e^2e_{,}\cos(2\tau-2\xi+\xi_{,})$$

The integral of which is

$$\left\{\frac{35}{4}m+\frac{145}{16}m^2+\frac{86407}{768}m^3\right\}e^2e_{,}\sin(2\tau-2\xi-\xi)$$

$$\pm\left\{-\frac{15}{4}m+\frac{135}{16}m^2+\frac{71231}{256}m^3\right\}e^2e_{,}\sin(2\tau-2\xi+\xi)$$

This is to be added to $\frac{2d.r\delta r-dr\delta r}{dt}$, p. 74, which gives

$$\left\{\left\{\frac{35}{4}-\frac{35}{16}=\frac{105}{16}\right\}m+\left\{\frac{145}{16}+27=\frac{577}{16}\right\}m^2\right.$$

$$\left.+\left\{\frac{446567}{3072}+\frac{86407}{768}=\frac{792195}{3072}\right\}m^4\right\}e^2e_{,}\sin(2\tau-2\xi-\xi)$$

$$+\left\{\left\{-\frac{15}{4}+\frac{15}{16}=-\frac{45}{16}\right\}m+\left\{\frac{135}{16}-\frac{77}{32}=\frac{193}{32}\right\}m^2\right.$$

$$\left.+\left\{\frac{11669}{3072}+\frac{71231}{256}=\frac{86441}{3072}\right\}m^4\right\}e^2e_{,}\sin(2\tau-2\xi+\xi)$$

This has to be divided by h or multiplied $1+\frac{m^2}{3}$, finally $\delta\lambda$ contains the terms,

$$\left\{\frac{105}{16}m+\frac{577}{16}m^2+\frac{266305}{1024}m^3\right\}e^2e_{,}\sin(2\tau-2\xi-\xi_{,}) \qquad \text{Arg. 24.}$$

$$+\left\{-\frac{45}{16}m+\frac{193}{32}m^2+\frac{863561}{3072}m^3\right\}e^2e_{,}\sin(2\tau-2\xi+\xi_{,}) \qquad \text{Arg. 27.}$$

The two former terms are identical with those determined by M. de Pontécoulant and by M. Plana, the term multiplied by m^3 has not been calculated by M. Plana, and my value does not agree with that formerly given by M. de Pontécoulant in consequence of the omissions already noticed. But M. de Pontécoulant has, at my request, recalculated these terms, and has (since p. 46 went to press) obtained coefficients agreeing with mine. He has found

instead of $\frac{359545}{1024}m^3e^2e_{,}$, vol. iv. p. 578; $\frac{259585}{1024}m^3e^2e_{,}$ and

" $\frac{196739}{1536}m^3e^2e_{,}$, " $\frac{872201}{3072}m^3e^2e_{,}$

The numerical conversion gives the following results:

	Arg. 24.	Arg. 27.
1st term,	$+5''.093(4)$	$-2''.183(4)$
2nd "	$+2.093(5)$	$+\ .350(5)$
3rd "	$+1.101(6)$	$+1.232(6)$
Total	8.287	$-\ .601$

* M. de Pontécoulant has examined again the coefficient of m^4 in these coefficients, and has furnished me with the following values :

Arg. 24, $\frac{179393}{384}m^4$, Arg. 27, $\frac{66823069}{18432}m^4$.

These numbers converted give

$$''.152(7)+.7 \text{ ind.}$$
$$1.178(7)+.9 \text{ ind.}$$

$$\text{Arg. 24, } ''.152(7)+\ .''7 \text{ ind.} +8''.287=9''.139$$
$$\text{Arg. 27, } 1.178(7)+\ .9 \text{ ind.} -\ .601=1.477$$

The coefficient of Arg. 27 is still different from $2''.2$, the value of Mr. Longstreth.

M. Plana has $-1''{\cdot}495$, Arg. 27, the result appearing to agree with that of M. de Pontécoulant, but this is in consequence of a mistake in the reduction ; M. Plana should have $\cdot''350(5)$, and then his coefficient would be $-1''{\cdot}749$.

I will now endeavour to obtain the coefficient of $e^4 e_, \sin(2\tau-2\xi-\xi_,)$ and $e^4 e_, \sin(2\tau-2\xi+\xi_,)$ in the expression for the longitude.

$$r=\mathrm{r}-\mathrm{r}^2\delta\frac{1}{r}$$

$$\delta\frac{1}{r}=\left\{\frac{35}{8}m-\frac{315}{64}me^2\right\}ee_,\cos(2\tau-\xi-\xi_,)$$

$$+\left\{-\frac{15}{8}m+\frac{945}{128}me^2\right\}ee_,\cos(2\tau-\xi+\xi_,)$$

$$+\frac{35}{4}me^2e_,\cos(2\tau-\xi_,)-\frac{15}{4}me^2e_,\cos(2\tau+\xi_,)-\frac{245}{64}me^3e_,\cos(2\tau-3\xi-\xi_,)$$

$$+\frac{525}{64}me^3e_,\cos(2\tau-3\xi+\xi_,)$$

$$\delta r=\left\{-\frac{35}{8}m+\frac{315}{64}me^2\right\}ee_,\cos(2\tau-\xi-\xi_,)$$

$$+\left\{\frac{15}{8}m-\frac{945}{128}me^2\right\}ee_,\cos(2\tau-\xi+\xi_,)$$

$$-\frac{35}{4}me^2e_,(2\tau+\xi_,)+\frac{15}{4}me^2e_,\cos(2\tau+\xi_,)+\frac{315}{64}me^3e_,\cos(2\tau-3\xi+\xi_,)$$

$$-\frac{555}{64}me^3e_,\cos(2\tau-3\xi+\xi_,)$$

$$\mathrm{dr}=\left(e-\frac{3e^3}{8}\right)\sin\xi+e^2\sin2\xi$$

d.r δr contains no term of the order we are seeking,

$$\mathrm{dr}\,\delta r=\left\{-\frac{315}{128}-\frac{105}{128}+\frac{35}{8}-\frac{315}{128}=-\frac{175}{128}\right\}me^4e_,\sin(2\tau-2\xi-\xi_,)$$

$$+\left\{-\frac{945}{256}-\frac{45}{128}+\frac{15}{8}+\frac{555}{128}=-\frac{405}{256}\right\}me^4e_,\sin(2\tau-2\xi+\xi_,)$$

By the theorem given in p. 60, as the coefficient of e^4 is zero in R, Arg. 9, the coefficients of e^4 in R, Args. 24 and 27, are also zero.

The coefficients of $\delta\lambda$, $\frac{105}{16}me^2e_,$ and $-\frac{45}{16}me^2e_,$ already found, have to be multiplied by $\frac{1}{h}=1+\frac{e^2}{2}$; hence, finally, the coefficients sought are,

$$\frac{105}{32}+\frac{175}{128}=\frac{595}{128}\text{ and }-\frac{45}{32}+\frac{405}{256}=\frac{45}{256}$$

These terms are insensible.

I will now endeavour to obtain the coefficients of $e^2e_,^3$.

$$\delta r=\frac{615}{64}mee_,^3\cos(2\tau-\xi-\xi_,)-\frac{15}{64}mee_,^3\cos(2\tau-\xi+\xi_,)$$

$$dr=e\sin\xi$$

$$dr\,\delta r=-\frac{615}{128}me^2e_,^3\sin(2\tau-2\xi-\xi_,)+\frac{15}{128}me^2e_,^3\sin(2\tau-2\xi+\xi_,)$$

$d.r\,\delta r$ contains no term of the order we are seeking.

By the theorem in p. 60,

$$\text{the coefficient of } e^2e_,^3\cos(2\tau-2\xi-\xi_,) \text{ in } R=\frac{123.21.8.5.15}{56.8.15.2.8}m^2=\frac{1845}{128}m^2 \, {}^*$$

$$e^2e_,^3\cos(2\tau-2\xi+\xi_,) \quad ,, \quad =-\frac{3.8.5.15}{8.8.15.2.8}m^2=-\frac{15}{128}m^2$$

The coefficients sought are,

$$\frac{615}{128}-\frac{615}{126}=0, \qquad \frac{15}{128}+\frac{15}{128}=\frac{15}{64}$$

These terms are insensible.

These examples illustrate the utility of the theorem in p. 60, the dovelopment of R, as far as it is carried in p. 30, is indeed now free from error, but as in the preceding instances, it may be wished to obtain higher terms which are not included in that development; and if, as in the preceding examples, they involve e and $e_,$, they may be obtained from the terms given in p. 30 without the labour of any calculation.

On the Construction of Lunar Tables.

As the object of those who have either calculated or have given methods intended to facilitate the calculation of the lunar inequalities, has been to procure tables of the moon founded upon theory alone, and to rescue this great branch of science from empiricism, I will endeavour now to show how far this object has been attained, and that it is possible to construct tables founded upon the coefficients of M. de Pontécoulant, that is, upon theory alone, far better than the empirical tables of Burckhardt so long in use.

In this enquiry we receive assistance from a quarter from which we did not expect it,

Via prima salutis,
Quod minimè reris, Graiâ pandetur ab urbe.

* These coefficients agree with those given by M. Delaunay, vol. i. p. 36.

We are indebted to the Americans, and especially to Prof. Peirce, for tables founded upon theory (although this is not the aspect under which they were presented); the coefficients which differ from those of M. de Pontécoulant and M. Plana being only three in number in the longitude, and the expressions for the latitude and the parallax being entirely due to M. Plana, and therefore due to theory alone. Although, at the time these Tables were published, these three coefficients deviated from the theoretical value assigned to them by M. de Pontécoulant, M. de Pontécoulant has lately reconsidered them, and the discrepancies are now almost, if not entirely removed.

These Tables founded on theory are invaluable, now that the Astronomical Society has attached so much importance to empirical tables, and that we are threatened to be carried back to the time of Clairaut.

I have already had occasion to refer to the American Tables, and I will here merely repeat as much as is necessary to render intelligible what follows, omitting minor details.

The longitude coefficients of the American Tables are generally taken from Plana, with the exception of eleven furnished by Mr. Longstreth of Philadelphia.

In eight* out of these eleven coefficients, Mr. Longstreth obtained empirically, and confirmed, the values due to M. de Pontécoulant.

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* The Americans attribute these coefficients to M. Longstreth, which had long previously been obtained with great labour and published by M. de Pontécoulant; but to attribute results to one philosopher which have previously been published by another, is an act of injustice and spoliation, which should be condemned by all who have at heart the interests of science. By also attributing other coefficients to the Astronomer Royal which they might equally have derived from M. de Pontécoulant, and by taking two erroneous equations from Prof. Hansen, the Americans gave to their tables a false aspect, and afforded the Astronomer Royal the opportunity of deriding them:

"These tables are a patchwork (a very good one, I believe); but I should never think of attaching the slightest value to such compositions, in comparison with tables formed from one broad theory." The theory upon which the American Tables are founded is consistent in all its parts; it is, therefore, "one broad theory," while Prof. Hansen's are not formed from "one broad theory," unless empiricism and theory mean the same thing. There is no branch of science (certainly none so difficult or extensive as the Lunar Theory) which is not a "patchwork" in this sense of the word, that our knowledge of it is derived from the talent and exertions of more than one person.

The remaining three are the coefficients of the following arguments :

	Damoiseau 1828.	Pont. 1846.	Longstreth.	Pont. 1860.
$2\tau-2\xi-\xi_{,}$	$+''\cdot7$	$-''1\cdot3$	$+''2\cdot2$	$+''1\cdot5$.
$4\tau+\xi$	$+\ 1\cdot9$	$+\ 1\cdot3$	$+\ 1\cdot9$	$+\ 1\cdot7$.
$4\tau-2\xi-\xi_{,}$	$+\ 3\cdot0$	$+\ 1\cdot2$	$+\ 3\cdot0$	$+\ 2\cdot5$.

Since I began to print this Part, M. de Pontécoulant, at my request, has re-examined his calculations, and has given me the coefficients in the right-hand column, which are very close to those of Mr. Longstreth. What theory has accomplished is therefore exhibited by the average error of these tables, which is about the same as that of the tables of Prof. Hansen. The coefficients of M. de Pontécoulant which now differ from those used in the American Tables, are of two kinds :

1st. Those which differ from the coefficients introduced into the American Tables by Mr. Longstreth.

2ndly. Those which differ from the coefficients introduced into the American Tables from M. Plana.

The differences due to the former source are,

$$-''\cdot7\sin(2\tau-2\xi+\xi_{,})\ -\ \cdot5\sin(4\tau-2\xi-\xi_{,})$$

The differences due to the second are,

$$2''\cdot0\sin\xi_{,}-\cdot6\sin(2\tau-2\xi)-\cdot5\sin(2\tau-\xi_{,})-\cdot4\sin(\xi-\xi_{,})$$
$$+\cdot3\sin(2\tau-\xi+\xi_{,})-\cdot3\sin(4\tau-4\xi)-\cdot3\sin\tau+\cdot3\sin(4\tau-\xi).$$

The theory is indebted to Mr. Longstreth for the determination of his eleven coefficients; and although Mr. Longstreth has not favoured us with any details, his researches must have been very laborious, and of greater value than the unpretending character of his Memoir in the Philadelphian Transactions would imply. I think, if Mr. Longstreth could be induced to turn his attention to the American Tables once more, small as the average error now is, he would be able slightly to reduce it.

In order to compare the places furnished by the American *Nautical* Almanac, for which the American Tables are employed, with such as would be furnished by the coefficients of M. de Pontécoulant, Mr. Farley began by selecting observations on contiguous days free from any obvious anomaly, and thus he formed 29 groups which are indicated in Table IV. This table contains the difference between the observed R. A. and Dec., and that calculated by means of the American Tables. These were changed into differences of longitude and latitude by means of Table VI. If x is the coefficient of a small subsidiary equation, A the sine of the argument, so that Ax is the

equation, l=A—O, the apparent error of the tables, $Ax+l$ is the apparent error which would obtain if the subsidiary equation were employed, and

$$\Sigma\left\{Ax+l\right\}^2$$

is the sum of the squares of the errors. In order that this may be a minimum, we have to determine x, the equation

$$x=-\frac{\Sigma(Al)}{\Sigma(A^2)}$$

In this way I found that to reduce the sum of the squares of the errors to a minimum in these groups, the coefficient Arg. 27 must be reduced by ″·7, making it 1″·5, which is identical with M. de Pontécoulant's last determination (see p. 80), and 2″·0 must be added to the annual equation, making it —″668·3. According to Le Verrier, the value of e' for 1857 is ·01676810. M. de Pontécoulant has employed the value ·01679182 (see p. 36); and therefore his value of the annual equation, which was —″668·932, requires alteration.

I noticed, in p. 39, a discrepancy in the term Arg. 5, multiplied by $m^3e_{\prime}\gamma^2$, between the coefficients of M. Plana and M. de Pontécoulant. Mr. Adams has verified M. Plana's value, and traced the error in M. de Pontécoulant's work. M. de Pontécoulant, with the proper value of $e_{\prime}$, and also by means of the removal of this error, now finds for the coefficient of the annual equation

—″668·268

for the year 1857, which is identical with the value which I have deduced from Mr. Farley's groups.

The annual change of the annual equation due to the variation of $e_{\prime}$ is ″·01688. Mr. Adams called my attention to this circumstance.

The labours which Mr. Farley has kindly undertaken for me in these determinations is very considerable, that is in finding the correction which will reduce to a minimum the sum of the squares of the errors; but, although the error of the American tables is very minute, I have no doubt that by extending the same process to other, or even to all the arguments, that error would be diminished. The machinery for this purpose here given is complete, and considering the importance to navigation of the Lunar Tables, the trouble would be repaid. The errors of the coefficients are so minute that I consider it would be sufficient to take them separrately, as we have done in the four cases

here given, without having recourse to simultaneous equations of condition between several variables.

The Astronomer Royal's last value of the parallactic inequality is not confirmed by our researches, as it appeared to increase the sum of the squares of the errors.

It would be very interesting to ascertain how much of l or A—O is due to the error of the tables, and how much to the error of observation. Although we are left in great measure to conjecture, we have some facts to guide us. The error of the tables must arise from one out of three sources. 1. An error in the longitude of the epoch. 2. An error in the mean motion. 3. Errors in the values of one or more of the coefficients employed. The first affects equally every place, and cannot produce any difference in the errors of different days. The mean motion is so well known, that an error of the mean motion can produce no sensible difference in the error of two consecutive days. Suppose there are 150 coefficients in the expression for the longitude which have an average error of $''\cdot 1$; suppose all these to have the same sign, and to be at their maximum together, the error of the longitude would be $''15\cdot 0$; but every argument is positive and negative exactly during the same time; if the period of any argument is a month, it will be positive for 15 days, and then negative for 15 days. Hence we may consider that the probability of any argument being positive upon any given day is precisely $\frac{1}{2}$; and it is easy to see, from the doctrine of chances, that the combinations in which the positive and negative signs are evenly distributed, or nearly so, will happen far oftener than the rest; so that events similar to the one contemplated above, in which the errors have not a tendency to destroy one another, may be entirely discarded from consideration. Moreover, as the average error is half the maximum error, this reduces the error at once to one-half of what it would be if the error were always at its maximum.

I think we are entitled to assume a very small quantity as the average error, because we are certain that there is no coefficient affected with a large error; and because there is no attempt, as in Burckhardt's Tables, to diminish the number of Arguments to be employed, by having recourse to an artificial expression which has no foundation in theory. If this be the case, whenever A—O differs much* on two

* The total quantity due to some of the arguments affected with the larger coefficients may differ considerably on consecutive days; but the error due to any given argument is very nearly the same.

consecutive days, this must be due to the error of observation. If the average error of the coefficients is

$''\cdot 1$, an error of $4''\cdot 0$ requires a preponderance of 40 coefficients,
$''\cdot 2$, „ „ „ 20 „

By preponderance, I mean the excess of the number of the + errors over the number of the — errors, or *vice-versâ*, arising from the coefficients. When the errors are considerable, I generally find a marginal note by the observer, "faint, scarcely visible," or the like, indicating the adverse circumstances under which the observation was made. Frequently when A—O is large, this circumstance is due to the observation having been made in broad daylight or in twilight. Mr. Longstreth noticed this circumstance. The tendency of these remarks is to show that the error due to the tables is much less variable in character than that due to the observations. The average error of the American Tables and of the observations combined for the three years that I have examined is $3''\cdot 2$. This is the average when all the imperfect observations made under adverse circumstances are retained.

I trust that M. de Pontécoulant will carry his approximations further, so as if possible to avoid altogether the necessity of having recourse to induction; but the accuracy of the American Tables is already such, that practically little advantage will be gained, unless the observations can be improved. If a slight change be made in one or more of the coefficients and places are calculated with them, a certain percentage of the places will be improved, even if the coefficients are made worse; if they are made better, and if several hundred places are taken into account, the majority of places will probably be improved, and the percentage will probably exceed 50. But this may not be the case, even if the sum of the squares of the errors be diminished; thus the substitution of M. de Pontécoulant's coefficient of $\sin(2\tau - \xi_{\prime})$ for that of M. Plana which is employed in the American Tables, reduced the sum of the squares of the errors from 5545·56 to 5490·64 upon 339 places; yet the majority of places were not improved, the percentage being only 46. Under these circumstances, and considering besides that M. de Pontécoulant and myself have no chance of fair play in this generation, there is little encouragement held out to us to make the enormous sacrifice of time and labour necessary to carry the approximations further. The determination of the correction x p. 85, by means of the groups is equivalent to the employment for this purpose of 132 picked observations; but the value

obtained was different from that obtained by all the observations of those years. Moreover, the observations of each year gave us a totally different result.* The Greenwich observatory furnishes about 100 transit observations per annum; their conformity with the places given by the American Almanac varies very much, in different years, if we may judge by the three years which I have examined. The squares of the errors of the places in the American *Nautical* Almanac were as follows, for the years 1856, 1857, and 1858.

	No. of Observations employed.	Longitude.		
		Σl^2	Average	Sq. rt.
1856	94	1496	16	4·0
1857	115	1438	12	3.5
1858	129	2612	20	4·5

These figures show how much the apparent errors of the places of the Moon in the American *Nautical* Almanac vary in different years; the places wide of the observations being far more numerous in 1858 than in the preceding years, although the observers and the system of observation, as far as can be gathered from the introduction to the observations, appear not to have varied. The observations of one year, although more than 100 in number, will not serve to determine the value of a coefficient within $''\!\cdot5$. Mr. Farley calculated the places with the Astronomer Royal's last value of the parallactic inequality; by this change, the sum of the squares of the errors was diminished for 1856 and 1857, and increased by a greater amount for 1858. The same thing happened with other coefficients which we changed; the places of one year were made better, and of another made worse.

These details show that Prof. Hansen's tables do not "far surpass in accuracy" † the American tables; but the difference is so slight, that

* Mr. Longstreth employed the observations of five years, 499 in all. I am anxious to know whether he found that different years gave him the same result, and whether or not he used equations of condition with more than one unknown quantity.

† This statement inserted in a resolution of the Board of Visitors of the Greenwich Observatory, was no doubt founded upon the announcement made to the Astronomical Society by the Astronomer Royal, that Professor

whether they give better places or not, can only be decided by a careful comparison of places calculated from both tables with the same observations.

According to Damoiseau, the secular mean motion

		°	′	″
of the mean longitude	=	307	52	41·6
of the perigee	=	109	2	46·6
of the node	=	—134	9	57·5

If we add 1336 circumferences to the first quantity, and divide by 36525, we find the daily motion

of the mean longitude = 13°·17639639.

If we add 11 circumferences to the second quantity, and divide by 36525, we find the daily motion

of the perigee = °·1114044153.

If we add 5 circumferences to the third quantity, and divide by 36525, we find the daily motion

of the node = —°·0529545783.

Hansen's Tables constituted the greatest stride ever made in practical science : "Probably in no recorded instance has practical science ever advanced so far in accuracy by a single stride." (*Monthly Notices*, April 8, 1859.)

In the Preface I have quoted the exact words of Prof. Hansen, as given in the *Monthly Notices*; the original is: "In meinen Mondtafeln habe ich einstweilen Coefficienten angewandt, die nicht frei von einigen Empirismus sind."

The correct translation appears to be: "In my Lunar Tables I have provisionally applied coefficients which are not free from some empiricism." (See Note by the Astronomer Royal in the *Monthly Notices* for Dec. 14, 1860.) The plain English of this passage is, that some of Prof. Hansen's coefficients were determined empirically.

The little that Prof. Hansen has allowed to transpire about the construction of his tables in the *Monthly Notices* is very obscure; but the following facts may, I think, be inferred: 1. That he multiplies all his coefficients in the expression for the longitude by empirical factors. 2. That he diminishes his latitudes empirically by 1″, to make them come right. 3. That his *factors of integration* were obtained empirically. 4. That some of his coefficients are altogether empirical. Tables constructed upon such a foundation, although they have been honoured with the approbation of the Visitors of the Greenwich Observatory and the medal of the Astronomical Society, are of a different complexion from those of the Americans.

The quantities employed in the American Tables for the mean motions are as follows:

of the mean longitude,	$13^{\circ}\cdot1763966914$
of the perigee,	$\cdot111404955236$
of the node,	$-\cdot0529537946$

The last quantities are taken from the Astronomer Royal's paper in vol. xvii. of the *Memoirs of the Royal Astronomical Society*, and they differ from Damoiseau's values in consequence of the introduction of the unfortunate equations of long period supposed to be due to the motion of Venus, which the Astronomer Royal then believed to be accurate. These equations have thrown into confusion this portion of the lunar theory; but as their present influence is counteracted by a corresponding alteration of the mean motion of the longitude, they will not for some years to come seriously affect the places of the moon given by the American Tables.

According to Prof. Hansen, the following are the secular motions (*Monthly Notices*, Nov. 10, 1854):

		°	′	″
of the mean longitude	=	307	53	39·61
of the perigee	=	109	3	2·46
of the node	=	—134	8	59·61.

But these are probably associated with Prof. Hansen's new equations of long period due to the action of Venus, one of which M. Delaunay has found to be insensible; in which case the mean motions will again have to be modified.

These numbers of Prof. Hansen give for the daily motions:—

of the mean longitude . . .	$13^{\circ}\cdot1763968$
of the perigee	$\cdot1114045$
of the node	$-\cdot0529542$

The following are the values for Greenwich, 1857, Jan. 0:—

With the constants used in the American Tables,

$$\tau=55^{\circ}\cdot0335 \quad \xi=310^{\circ}\cdot3971 \quad \xi_{\prime}=359^{\circ}\cdot6176 \quad \eta=324^{\circ}\cdot3105$$

With Prof. Hansen's constants,

$$\tau=55\cdot0243 \quad \xi=310\cdot4007 \quad \xi_{\prime}=359\cdot6243 \quad \eta=324\cdot3071$$

so that the cause of any discrepancy which may be found to exist between places given by Hansen's tables and those by the American tables must be sought elsewhere than in the arguments of the equations. These differences are considerable, amounting sometimes to $''12$.

The "form of the formula of the longitude is modified" in the American Tables by substituting for the term

$$-''411{\cdot}7 \sin 2\eta$$

depending upon the mean argument of the latitude η, the following term depending upon the true argument of the latitude, $\bar{\eta}$

$$-''416{\cdot}7 \sin 2\bar{\eta}, \text{ which should be } -''416{\cdot}9 \sin 2\bar{\eta}.$$

I believe the expression in the 2nd page of the introduction to be incorrect; and this alteration appears to have been the cause of a great deal of mischief without any corresponding advantage.* If I am not mistaken, some of the tables which involve η in the expression for the longitude will have to be recast, and new ones added. The Americans profess to take in quantities of the order $''{\cdot}2$; the following table shows the errors of the coefficients of the American Tables. It will be seen that there are no less than seven tables with erroneous coefficients; and three new tables are required for coefficients amounting to $''{\cdot}2$.

The new equations required, in their order of magnitude, are:

$$-1''{\cdot}5 \sin(2\tau-2\xi+2\eta) - 1''{\cdot}1 \sin(2\tau-\xi+2\eta) - ''{\cdot}3 \sin(3\xi+2\eta)$$

and the tables 16, 56, 28, 50, 54, 64, and 67 must be recast.

The first column gives the coefficient which ought to be found in the sixth page of the introduction to the American Tables. The second column contains Plana's coefficient, except in the three cases marked L, which the Americans attribute to M. Longstreth, but which are, in fact, due to M. de Pontécoulant. The sum of these is the correct coefficient which is given in the next column. The next column

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* The effect is chiefly to knock out the term of which the argument is $\xi+2\eta$, and to double that of which the argument is $\xi-2\eta$.

gives the coefficient actually employed by the Americans, and the error is in the one beyond.

Argument.			Plana.	True coefficients	American Tables.	Error.	No. of Table.
		″	″	″	″	″	
$2\xi+2\eta$		4·0	− 4·1	− ·1		− ·1	
$2\xi-2\eta$		− 1·0	− 1·1	− 2·1	− 2·0	·1	78
$\xi-2\eta$	[2001′] revd.	45·4	39·4 L	84·8	85·0	·2	16
$\xi+2\eta$	[2001]	45·4	−45·2	·2	− ·2	− ·4	56
$3\xi+2\eta$			− ·3	− ·3		·3	
$2\tau-\xi-2\eta$	[202′1] revd.	9·8	·5 L	10·3	9·8	− ·5	28
$2\tau-\xi+2\eta$		8·8	− 9·9 *	− 1·1		1·1	
$2\tau+\xi+2\eta$	[2021]	·9	− ·6	·3	− ·2	− ·5	50
$2\tau+\xi-2\eta$	[2′021]	− ·1	− 6·1	− 6·2	− 6·3	− ·1	34
$2\tau-2\xi-2\eta$	[202′1] revd.	1·4	− ·2	1·2	·2	− 1·0	54
$2\tau-2\xi+2\eta$		− ·6	− ·9	− 1·5		1·5	
$2\tau+2\eta$		5·7	− 5·8 L	− ·1		− ·1	
$2\tau-2\eta$	[202′0] revd.	3·7	54·9	58·6	58·7	·1	17
$\xi-\xi_,-2\eta$	[2101′] revd.	·3	·2	·5	·4	·1	55
$\xi+\xi_,-2\eta$	[2′101]	− ·2	− ·2	− ·4	− ·4		51
$4\tau-\xi-2\eta$	[204′1] revd.		·4	·4	·6	·2	64
$\tau-2\eta$	[201′0] revd.	− ·2	− ·7	− ·9	− 1·0	− ·1	57
$\xi_,+2\eta$	[2100]	− 1·3	·6	− ·7	− ·7		61
$\xi_,-2\eta$	[21′00] revd.	− 1·3	·1	− 1·2	− 1·3	− ·1	52
$2\tau-\xi-\xi_,-2\eta$	[212′1] revd.	·4	·1	·5	·4	·1	65
$2\tau-\xi-\xi_,+2\eta$		·4	− ·3	·1		− ·1	
$2\tau-\xi_,+2\eta$	[21′20]	·3	− ·2	·1	·3	·2	66
$2\tau-\xi_,-2\eta$	[212′0] revd.	·3	2·3	2·6	2·6		58
4η			·4	·4	·4		61
$\xi+\xi_,+2\eta$		− ·2	·2				
$\xi-\xi_,+2\eta$		·3	− ·2	·1		·1	
$2\tau-2\xi_,-2\eta$			·1	·1		·1	
$2\tau+2\xi-2\eta$	[2′022]		− ·5	− ·5	− ·5		53
$\tau+2\eta$		− ·2	·1	− ·1		·1	
$3\tau-2\eta$	[2′030]		− ·2	− ·2	− ·2		71
$2\tau+\xi-\xi_,-2\eta$	[2′1′21]		− ·3	− ·3	− ·3		69
$2\tau+\xi_,-2\eta$	[21′2′0] revd.		− 1·5	− 1·5	− 1·5		75

As I see no use in introducing the true argument of latitude, I have not calculated the coefficients here given with great care. I trust the Americans will carefully revise all this part of their work.

On the Calculation of an Ephemeris by Machinery.

It seems worth while to enquire whether it would not be better to

* This coefficient is incorrectly given in the Table inserted in the Appendix.

do away with the use of tables altogether for the small inequalities, and employ machinery for the purpose of obtaining them.

Fig. 1.

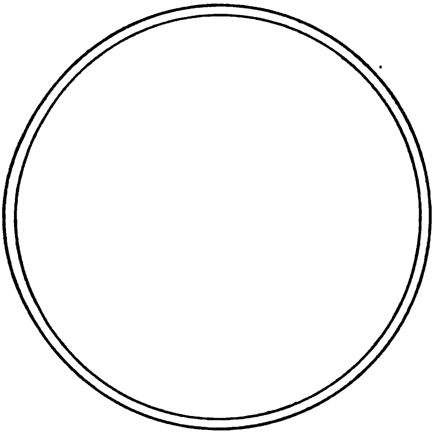

Fig. 2.

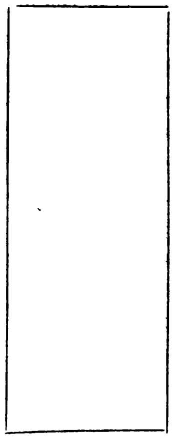

Fig. 3.

	19·8
	19·3
	18·6
	17·6
	16·4
	15·6
	13·4
	11·7
A.	10·0
	8·3
	6·6
	5·0
	3·6
	2·4
	1·4
	·7
	·2
	·0

Fig. 1 is the side view of a wheel.

Fig. 2 is the front view of the circumference of the same wheel, the *figures being omitted.*

Fig. 3 is the circumference drawn out, and therefore is to the last in height as, $\frac{3.14159}{2}$: 1. Equal divisions on this represent equal divisions on the circumference of the wheel. Now, suppose this wheel is intended to give the value of an equation, of which the epoch at the given time is 0, suppose the ephemeris required for every twelve hours, and suppose the coefficient of the argument to be ″10. Suppose, also, the period for the argument to be eighteen days. The circumference of the wheel should give at A, the value of the equation which is 0. Now, suppose the wheel moved through an angle of 10°, it should now indicate at A the value of the equation for 12 hours later, which is ″1·7, and so on. If ″10 is added throughout to get rid of the negative values, then the proper numbers will be those inserted in fig. 3. If

a roller be brought into contact with the wheel successively at A, the figures may be printed off upon it.

Now, suppose a second wheel moving round the same axis as the first, and representing another argument, with a different period, and carrying on its circumference the proper figures. Suppose the period to be double the period of the other, this wheel must be moved through 5°, while the former is moved through 10°. And so for any number of equations. The difficulties to be overcome consist in being able to make a number of wheels revolving side by side on a common axis have different periods of revolution corresponding to the different periods of the arguments, and in giving the roller a motion of rotation, and also a motion of *va et vient*. At each movement, the roller must move through a given space, and also be pressed against the circumference of the wheel carrying the figures. I think the longer equations must be computed as at present; but there are a great multitude of small equations under ″10 which might be obtained with great rapidity if this idea can be carried out in practice. I imagine that the wheels might have different velocities given them, by means of pinions of different sizes fixed upon a common axis. I have inserted the figures in Fig. 3 for every ten degrees, but without making the wheels inconveniently large they might be inserted for every degree. The results would appear on the roller in a horizontal line. If the numbers are placed vertically on the wheel, then they would come out over each other, but to get the same number of figures inserted, the wheel must be larger. The machine must be *set*, and then it would go on printing as long as the motive power continued.

Unless some method can be devised of computing with facility the sum of the smaller equations, a great deal of labour which has been taken to obtain the coefficients accurately will probably be thrown away. Probably more equations might be taken into account with advantage, which are now discarded on account of the trouble they would occasion. Such work would be far within the reach of Mr. Babbage's *Analytical Engine*, described by General Menabrea in the third volume of Taylor's Scientific Memoirs.

TABLES.

TABLE I.

Comparison of the Numerical Values of the Coefficients in the Expression for the Longitude of the Moon.

Lubbock. Arguments.	Nos.	Pontécoulant. Arguments.	Nos.	The Americans Argts.	Nos.	Bürg.	Burckhardt.	Damoiseau.	Plana.	Pontécoulant. P.	The Americans. A.	Autʸ.	P.—A.
						″	″	″	″	″	″		″
2τ	1	2ξ	30	[20]	3	2373·0	2373·4[b]	2370·00	2370·320	2370·799*	2371·0	A.	− ·2
ξ	2	ϕ	1	[1]	1	22638·2	22639·7	22639·70	22641·626	22639·700	22639·2	A.	+ ·5
$2\tau-\xi$	3	$2\xi-\phi$	31	[21′]	2	4588·2	4587·0	4589·61	4585·648	4586·999	4586·9	A.	+ ·1
$2\tau+\xi$	4	$2\xi+\phi$	32	[21]	7	192·5	192·6	192·22	192·146	192·142	192·1	P.	·0
$\xi_,$	5	ϕ'	6	[100]	4	−674·2	−673·3[b]	−673·70	−668·644	−668·932	−670·3	A.	+1·4
$2\tau-\xi_,$	6	$2\xi-\phi'$	33	[1′20]	8	167·0	166·8[b]	165·56	165·850	165·437	165·9	P.	− ·5
$2\tau+\xi_,$	7	$2\xi+\phi'$	34	[120]	15	− 27·3	− 27·3[b]	− 24·82	− 23·611[a]	− 25·036	− 25·0	L.	·0
2ξ	8	2ϕ	2	[2]	1	768·3	768·3	768·72	769·477	769·533	769·5	P.	·0
$2\tau-2\xi$	9	$2\xi-2\phi$	35	[22′]	5	211·6	212·2	211·57	212·363	211·787	212·4	P.	− ·6
$2\tau+2\xi$	10	$2\xi+2\phi$	36	[22]	16	14·8	14·7	14·74	14·119	14·109	14·1	P.	·0
$\xi+\xi_,$	11	$\phi+\phi'$	9	[101]	10	−111·3	−109·9	−109·27	−111·099[a]	−109·886	−110·0	L.	+ ·1
$2\tau-\xi-\xi_,$	12	$2\xi-\phi-\phi'$	37	[1′21′]	6	206·2	206·3	207·09	209·742[a]	206·919	206·9	L.	·0
$2\tau+\xi+\xi_,$	13	$2\xi+\phi+\phi'$	40	[121]	25	− 2·9	− 2·9	− 3·03	− 2·884	− 2·876	− 2·9	P.	·0
$\xi-\xi_,$	14	$\phi-\phi'$	8	[1′01]	9	148·1	147·5	147·74	148·059[a]	147·691	148·1	P.	− ·4
$2\tau-\xi+\xi_,$	15	$2\xi-\phi+\phi'$	38	[121′]	14	− 27·4	− 27·6	− 28·67	− 28·811	− 28·511	− 28·8	P.	+ ·3
$2\tau+\xi-\xi_,$	16	$2\xi+\phi-\phi'$	39	[1′21]	18	14·6	13·5	14·69	14·044	14·000	14·0	P.	·0
$2\xi_,$	17	$2\phi'$	7	[200]	4	− 6·0	− 7·3	− 7·34	− 7·872	− 7·851	− 7·9	P.	·0
$2\tau-2\xi_,$	18	$2\xi-2\phi'$	41	[2′20]	22	7·8	7·8	7·92	7·813	7·790	7·8	P.	·0
$2\tau+2\xi_,$	19	$2\xi+2\phi'$	42				− ·1	− ·33	− ·065	− ·065			·0
3ξ	20	3ϕ	3	[3]	1	36·4	36·2	36·94	36·720	36·719	36·7	P.	·0
$2\tau-3\xi$	21	$2\xi-3\phi$	43	[23′]	19	14·0	14·0[b]	12·81	12·807	12·899	12·8	P.	+ ·1
$2\tau+3\xi$	22	$2\xi+3\phi$	44	[23]	57		1·4	1·27	3·309[a]	·902	·8	P.	+ ·1
$2\xi+\xi_,$	23	$2\phi+\phi'$	11	[102]	24	− 7·6	− 6·5	− 7·67	− 7·344	− 7·347	− 7·3	P.	·0
$2\tau-2\xi-\xi_,$	24	$2\xi-2\phi-\phi'$	45	[1′22′]	21	8·6	9·1	8·99	7·762[a]	9·161	9·2	L.	·0
$2\xi-\xi_,$	26	$2\phi-\phi'$	10	[1′02]	20	9·4	9·1	9·74	9·644	9·651	9·6	P.	·0
$2\tau-2\xi+\xi_,$	27	$2\xi-2\phi+\phi'$	46	[122′]	69	2·5	2·4[b]	2·55	− 1·842	− ·370	2·2	L.	−2·6
$2\tau+2\xi-\xi_,$	28	$2\xi+2\phi-\phi'$	47	[1′22]	43		·8	·90	·607	·604	·6	P.	·0
$\xi+2\xi_,$	29	$\phi+2\phi'$	13	[201]	35		− 1·2	− 1·14	− 1·172	− 1·167	− 1·2	P.	·0
$2\tau-\xi-2\xi_,$	30	$2\xi-\phi-2\phi'$	49	[2′21′]	28	6·1	7·2	7·52	7·527	7·482	7·5	P.	·0
$\xi-2\xi_,$	32	$\phi-2\phi'$	12	[2′01]	33	1·2	1·2[b]	2·52	2·131	2·343	2·1	P.	+ ·2
$2\tau-\xi+2\xi_,$	33	$2\xi-\phi+2\phi'$	50	[221′]	34	− 3·3	− 1·0[b]	− 2·59	− 2·014	− 2·027	− 2·0	P.	·0
$2\tau+\xi-2\xi_,$	34	$2\xi+\phi-2\phi'$	51	[2′21]	42	·6	·4	·52	·323	·321	·3	P.	·0

4ξ	38	4ϕ		[4]	1	1·8	1·8	1·99	2·003	1·980	2·0	P.	·0
$2\tau-4\xi$	39	$2\xi-4\phi$	52				1·1	·93	·873[a]	·606			− ·3
$3\xi+\xi_{,}$	41	$3\phi+\phi'$		[103]	41		− ·2	− ·39	− ·365	− ·362	− ·4	P.	·0
$2\tau-3\xi-\xi_{,}$	42	$2\xi-3\phi-\phi'$					·5	·48	·328	·325			·0
$3\xi-\xi_{,}$	44	$3\phi-\phi'$	16	[1'03]	40		·4	·57	·365	·362	·4	P.	·0
$2\tau-3\xi+\xi_{,}$	45	$2\xi-3\phi+\phi'$	54				− ·2		− ·140	− ·139			·0
$2\tau-2\xi-2\xi_{,}$	48	$2\xi-2\phi-2\phi'$	55				·4	·11	·157	·156			·0
2η	62	2η	18			−411·5	−412·3	−411·67	−411·041	−411·624			− ·6
$2\tau-2\eta$	63	$2\xi-2\eta$	57			56·7	56·3[b]	54·83	54·915	55·109			+ ·2
$2\tau+2\eta$	64	$2\xi+2\eta$				− 6·0	− 5·8	− 5·75	− 3·376[a]	− 5·728	− 5·8	L.	+ ·1
$\xi-2\eta$	65	$\phi-2\eta$	19			38·8	38·7	39·51	37·191[a]	39·427	39·4	L.	·0
$\xi+2\eta$	66	$\phi+2\eta$	20			− 45·2	− 45·1	− 45·12	− 45·201	− 45·105			+ ·1
$2\tau-\xi-2\eta$	67	$2\xi-\phi-2\eta$	59				·3	·27	·030	·514	·5	L.	·0
$2\tau-\xi+2\eta$	68	$2\xi-\phi+2\eta$				− 10·6	− 9·5	− 9·65	− 9·384	− 9·221			+ ·2
$2\tau+\xi-2\eta$	69	$2\xi+\phi-2\eta$	61			− 6·9	− 5·6	− 6·65	− 6·151	− 6·065			+ ·1
$2\tau+\xi+2\eta$	70	$2\xi+\phi+2\eta$					− ·9	− 1·00	− ·626	− ·619			·0
$\xi'-2\eta$	71	$\phi'-2\eta$					1·3[b]	·04	·130	·130			·0
$\xi'+2\eta$	72	$\phi'+2\eta$					1·3[b]	·37	·585	·636			·0
$2\tau-\xi_{,}-2\eta$	73	$2\xi-\phi'-2\eta$	63				1·5[b]	2·92	2·329	2·306			·0
$2\tau-\xi_{,}+2\eta$	74	$2\xi-\phi'+2\eta$					− ·3	− ·17	− ·189	− ·187			·0
$2\tau+\xi_{,}-2\eta$	75	$2\xi+\phi'-2\eta$						− 1·34	− 1·475	− 1·460			·0
$2\xi-2\eta$	77	$2\phi-2\eta$					− 1·9	− 1·30	− 1·079	− 1·405			− ·3
$2\xi+2\eta$	78	$2\phi+2\eta$	24				− 4·0	− 3·97	− 4·089	− 4·037			·0
$2\tau-2\xi-2\eta$	79	$2\xi-2\phi-2\eta$	67				− ·8	− ·52	− ·177	− ·174			·0
$2\tau-2\xi+2\eta$	80	$2\xi-2\phi+2\eta$					− ·4	− ·60	− ·941	− ·697			+ ·2
$2\tau+2\xi-2\eta$	81	$2\xi+2\phi-2\eta$	69				− ·1	− ·52	− ·529	− ·523			·0
$\xi+\xi_{,}-2\eta$	83	$\phi+\phi'-2\eta$	27				·2		− ·162	− ·160			·0
$\xi+\xi_{,}+2\eta$	84	$\phi+\phi'+2\eta$	28				·2		·195	·192			·0
$\xi-\xi_{,}-2\eta$	89	$\phi-\phi'-2\eta$	25				− ·3		·162	·160			·0
$\xi-\xi'+2\eta$	90	$\phi-\phi'+2\eta$	26				− ·3		− ·195	− ·192			·0
τ	101	ξ	70	[10]	3	−122·9	−123·5[b]	−122·48	−122·110	−122·378	−122·1	P.	− ·3
$\tau-\xi$	102	$\xi-\phi$	71	[11']	5	− 16·1	− 18·2	− 17·19	− 18·045	− 17·795	− 18·0	P.	+ ·2
$\tau+\xi$	103	$\xi+\phi$	72	[11]	16	− 9·3	− 8·8	− 8·40	− 8·237	− 8·365	− 8·2	P.	− ·2

* The Americans have, unfortunately, modified the form of the formula of the longitude, by substituting the terms dependent upon the true arguments of the latitude, instead of the equivalents. The differences in the last column are the coefficients which may be used in subsidiary tables, to bring the places from the American Tables to what they would have been if Pontécoulant's coefficients had been employed.

Against the coefficient used in the American Tables, is the initial letter of the authority for the corresponding coefficient, whether PLANA, AIRY, or LONGSTRETH.

NOTES BY M. DE PONTÉCOULANT, VOL. IV. P. 624.

* Ce coefficient est une des arbitraires de la théorie; nous avons cru devoir adopter le résultat de Burckhardt.

(a) Nous avons marqué d'un [a] les resultats de M. Plana que nous avons reconnus fautifs par la révision de ses formules.

(b) La lettre [b] indique les résultats dérivés des Tables de Burckhardt, qui ont évidemment besoin de correction.

Comparison of the Numerical Values of the Coefficients in the Expression for the Longitude, etc.—*Continued.*

Lubbock.		Pontécoulant.		The Americans		Bürg.	Burckhardt.	Damoiseau.	Plana.	Pontécoulant. P.	The Americans. A.	Autʸ.	P.—A.
Arguments.	Nos.	Arguments.	Nos.	Argts.	Nos.								
$\tau-\xi_{,}$	104	$\xi-\phi'$	73	[1'10]	22	2·5	2·0[b]	− 0·48	− ·383	− ·378	− ·4	P.	·0
$\tau+\xi_{,}$	105	$\xi+\phi'$	74	[110]	17	13·8	13·9[b]	17·56	17·216	17·252	17·2	P.	·0
$\tau-2\xi$	106	$\xi-2\phi$	75	[12']	27		− 1·7	− 1·17	− ·796	− ·794	− ·8	P.	·0
$\tau+2\xi$	107	$\xi+2\phi$	76	[12]	38		− ·3	− ·48	− ·357	− ·356	− ·4	P.	·0
$\tau-\xi-\xi_{,}$	108	$\xi-\phi-\phi'$	77				− ·2	− ·29	− ·025	− ·025			·0
$\tau+\xi+\xi_{,}$	109	$\xi+\phi+\phi'$	79	[111]	36		·7	1·25	·994	·991	1·0	P	·0
$\tau-\xi+\xi_{,}$	110	$\xi-\phi+\phi'$	78				·7	2·05	·466	·388			− ·1
3τ	116	3ξ	80	[30]	3		2·7[b]	·01	·887	·887	0·9	P.	·0
$3\tau-\xi$	117	$3\xi-\phi$	81	[31']	31		− 2·1	− 3·04	− 2·950	− 2·948	− 3·0	P.	·0
$3\tau+\xi$	118	$3\xi+\phi$	82				·6	·33	·143	·144			·0
$3\tau-2\xi$	121	$3\xi-2\phi$	85	[32']	39		− ·2		− ·640	− ·638	− ·6	P.	·0
$3\tau-\xi+\xi_{,}$	125	$3\xi-\phi+\phi'$					·2		·194	·210			·0
4τ	131	4ξ		[40]	3	14·3	16·1	14·85	14·409	14·245	14·4	P.	− ·2
$4\tau-\xi$	132	$4\xi-\phi$		[41']	13	40·7	38·6	38·62	38·001	38·252	38·0	P.	+ ·3
$4\tau+\xi$	133	$4\xi+\phi$		[41]	26	1·9	2·3[b]	− ·43	·855[a]	1·731	1·9	L.	− ·2
$4\tau-\xi_{,}$	134	$4\xi-\phi'$		[1'40]	54		2·0[b]	·82	1·210	1·282	1·2	P.	+ ·1
$4\tau+\xi_{,}$	135	$4\xi+\phi'$		[140]	65		− ·2	− ·09	− ·201	− ·224	− ·2	P.	·0
$4\tau-2\xi$	136	$4\xi-2\phi$		[42']	2	31·4	31·4	31·19	34·518[a]	31·153	31·2	L.	·0
$4\tau+2\xi$	137	$4\xi+2\phi$					− ·1	·73		·139			
$4\tau-\xi-\xi_{,}$	138	$4\xi-\phi-\phi'$		[1'41']	30	3·4	4·1	3·32	3·837	3·824	3·8	P.	·0
$4\tau-\xi+\xi_{,}$	140	$4\xi-\phi+\phi'$		[141']	55		− ·7	− ·49	− ·922	− ·746	− ·9	P.	+ ·1
$\xi-4\eta$		$\phi-4\eta$					− ·2		− ·140	− ·137			·0
$4\tau-3\xi$		$4\xi-3\phi$		[43']	37		·7	1·39	·502	·432	·5	P.	− ·1
$4\tau-2\xi-\xi_{,}$		$4\xi-2\phi-\phi'$		[1'42']	32		2·3[b]	3·05	1·197	2·151	3·0	L.	− ·9
$4\tau-2\xi+\xi_{,}$		$4\xi-2\phi+\phi'$		[142']	58		− ·2	− ·50	− ·513	− ·510	− ·5	P.	·0
$4\tau-2\eta$		$4\xi-2\eta$					·5	·64	·054	·054			·0
$4\tau-\xi-2\eta$		$4\xi-\phi-2\eta$					·6	·39	·397	·393			·0
$4\tau+\xi-2\eta$		$4\xi+\phi-2\eta$					− ·1		− ·036	− ·036			·0
$6\tau-\xi$		$6\xi-\phi$					·2			·151			
$6\tau-2\xi$		$6\xi-2\phi$					1·0[b]	·53		·256			

Notes by M. de Pontécoulant, vol. iv. p. 624. (*a*) Nous avons marqué d'un [a] les résultats de M. Plana que nous avons reconnus fautifs par la révision de ses formules. (*b*) La lettre [b] indique les résultats dérivés des Tables de Burckhardt, qui ont évidemment besoin de correction.

Notes by Sir J. Lubbock. Args. 27, 133, and $4\tau-2\xi-\xi_{,}$. The numbers here attributed to M. de Pontécoulant are not those published by him. M. de Pontécoulant has recently carried the approximation further, and kindly communicated to me the values here given.
According to the last determination of the Astronomer Royal from the observations, the coefficient of sin ξ is 22639″·06.

TABLE II.

Comparison of the Numerical Values of the Coefficients of the Periodic Inequalities of the Latitude of the Moon, in Terms of her Mean Longitude.

Lubbock. Argument.	No.	Pontécoulant. Argument.	No.	Bürg.	Burckhardt.	Damoiseau.	Plana.	Pontécoulant.	Pontécoulant. — Plana.
				″	″	″	″	″	″
η	146	η	105	18465·2	18461·700	18465·4	18465·538	18461·700*	−3·8
$2\tau-\eta$	147	$2\xi-\eta$	130	623·6	623·4	622·1	621·476	623·569	+2·1
$2\tau+\eta$	148	$2\xi+\eta$	131	117·0	116·9	117·3	117·580	117·512	− ·1
$\xi-\eta$	149	$\phi-\eta$	108	999·5	993·5[a]	999·5	1000·509	1000·664	+ ·1
$\xi+\eta$	150	$\phi+\eta$	109	1011·4	1009·5	1010·5	1010·894	1011·198	+ ·3
$2\tau-\xi-\eta$	151	$2\xi-\phi-\eta$	132	165·9	165·8	166·0	166·615	166·649	·0
$2\tau-\xi+\eta$	152	$2\xi-\phi+\eta$	133	199·9	198·6	199·8	196·224	198·810	+2·6
$2\tau+\xi-\eta$	153	$2\xi+\phi-\eta$	134	33·4	34·5	34·3	35·039[1]	33·445	−1·6
$2\tau+\xi+\eta$	154	$2\xi+\phi+\eta$	135	15·1	14·9	15·1	14·774	14·975	+ ·2
$\xi_,-\eta$	155	$\phi'-\eta$	110		− 4·2		− 4·167	− 4·221	·0
$\xi_,+\eta$	156	$\phi'+\eta$	111		− 5·9	− 6·4	− 5·448	− 5·595	− ·1
$2\tau-\xi_,-\eta$	157	$2\xi-\phi'-\eta$	136	29·9	31·0	29·8	29·535	29·650	+ ·1
$2\tau-\xi_,+\eta$	158	$2\xi-\phi'+\eta$	137		7·5	8·1	7·593	7·669	+ ·1
$2\tau+\xi_,-\eta$	159	$2\xi+\phi'-\eta$	138	11·4	− 12·1	− 12·2	− 12·272	− 11·584	+ ·7
$2\tau+\xi_,+\eta$	160	$2\xi+\phi'+\eta$	139		− 1·3	− 1·4	− 1·194	− 1·426	− ·2
$2\xi-\eta$	161	$2\phi-\eta$	112	32·3	33·7	31·8	32·343	32·108	− ·2
$2\xi+\eta$	162	$2\phi+\eta$	113	61·9	62·4	61·9	62·462	62·422	·0
$2\tau-2\xi-\eta$	163	$2\xi-2\phi-\eta$	140	16·6	14·8	15·5	16·217[2]	15·021	−1·2
$2\tau-2\xi+\eta$	164	$2\xi-2\phi+\eta$	141		− 1·6		− 1·883	− 1·849	·0
$2\tau+2\xi-\eta$	165	$2\xi+2\phi-\eta$	142		1·9		2·377	2·159	− ·2
$2\tau+2\xi+\eta$	166	$2\xi+2\phi+\eta$	143		·8		·867	1·285	+ ·4
$\xi+\xi_,-\eta$	167	$\phi+\phi'-\eta$	116		− 8·1[a]		− 4·864	− 4·847	·0
$\xi+\xi_,+\eta$	168	$\phi+\phi'+\eta$	117		− 5·2	− 5·3	− 5·305	− 5·253	·0
$2\tau-\xi-\xi_,-\eta$	169	$2\xi-\phi-\phi'-\eta$	144		8·8	− 8·4	8·241	8·311	+ ·1
$2\tau-\xi-\xi_,+\eta$	170	$2\xi-\phi-\phi'+\eta$	145		8·7		9·226	8·875	− ·4
$2\tau+\xi+\xi_,-\eta$	171	$2\xi+\phi+\phi'-\eta$	150		− ·6		− ·905[3]	− ·739	+ ·2
$\xi-\xi_,-\eta$	173	$\phi-\phi'-\eta$	114		6·1		5·183	5·018	− ·2
$\xi-\xi_,+\eta$	174	$\phi-\phi'+\eta$	115		6·9		6·360	6·313	·0
$2\tau-\xi+\xi_,-\eta$	175	$2\xi-\phi+\phi'-\eta$	146		− 1·4		− 1·945	− 1·978	·0
$2\tau-\xi+\xi_,+\eta$	176	$2\xi-\phi+\phi'+\eta$	147		− ·7[a]		− 2·692	− 2·675	·0
$2\tau+\xi-\xi_,-\eta$	177	$2\xi+\phi-\phi'-\eta$	148	1·7	1·9	1·3	1·576	1·564	·0
$2\tau+\xi-\xi_,+\eta$	178	$2\xi+\phi-\phi'+\eta$	149	1·1	·6	1·1	·588	·583	·0
$2\xi_,-\eta$	179	$2\phi'-\eta$	118	− ·1	− ·3		− ·106	− ·105	·0
$2\xi_,+\eta$	180	$2\phi'+\eta$	119	·1	− ·3		− ·146	− ·145	·0

Notes by M. de Pontécoulant, vol. iv. p. 631.

* Ce coefficient est une des arbitraires de la théorie; nous avons cru devoir adopter le résultat de Burck-rdt.

(1) La différence entre le résultat de M. Plana et le nôtre, tient à une faute qui existe dans sa formule ana-tique, et qui tombe sur les termes de l'ordre $m^3e\gamma$.

(2) La différence de 1″ entre le résultat de M. Plana et le nôtre, tient à une erreur dans la réduction de s formules; au lieu de 4″·879(5) (t. I. p. 720), on doit lire 3″·879(5) dans le coefficient de sin $(2\xi-2\phi-\eta)$.

(3) Il y a erreur de signe dans la formule de M. Plana; la réduction en nombres est d'ailleurs également ative.

(a) Les résultats marqués d'un [a] dans ce tableau ont évidemment besoin de correction.

COMPARISON OF THE NUMERICAL VALUES, ETC.—*Continued.*

Lubbock. Argument.	No.	Pontécoulant. Argument.	No.	Bürg.	Burckhardt.	Damoiseau.	Plana.	Pontécoulant.	Pontécoulant — Plana.
				″	″	″	″	″	″
$2\tau-2\xi_{,}-\eta$	181	$2\xi-2\phi'-\eta$	152	1·0	·3	1·1	·627[4]	·933	+ ·3
$2\tau-2\xi_{,}+\eta$	182	$2\xi-2\phi'+\eta$	153	·3	·3	·4	·172	·171	·0
$2\tau+2\xi_{,}-\eta$	183	$2\xi+2\phi'-\eta$	154	− ·2		− ·1	− ·062		
$\tau-\eta$	185	$\xi-\eta$	108	− 5·4	− 5·4	− 4·6	− 4·618	− 4·129	+ ·5
$\tau+\eta$	186	$\xi+\eta$	109	− 5·5	− 5·5	− 5·3	− 5·016	− 5·064	·0
$3\xi-\eta$	195	$3\phi-\eta$	120	·8	3·7[a]	1·7	1·687	1·669	·0
$3\xi+\eta$	196	$3\phi+\eta$	121	4·0	2·2[a]	4·0	4·085	4·043	·0
3η		3η		− 5·5	5·7	− 6·2	− 6·424	− 6·397	·0
$\xi-3\eta$		$\phi-3\eta$		− 2·9	3·8	− 2·8	2·848	2·786	− ·1
$2\tau-3\xi-\eta$		$2\xi-3\phi-\eta$	155	− 1·9	0·0[a]	− 2·1	1·075	·950	− ·1
$2\tau-3\xi+\eta$		$2\xi-3\phi+\eta$	156	− ·3	·6	·3	·215	·213	·0
$2\tau-3\eta$		$2\xi-3\eta$		− 2·6	2·7	2·1	2·384	2·318	− ·1
$2\tau+3\eta$		$2\xi+3\eta$		− ·1		− ·1	·241	·238	·0
$2\tau-\xi-3\eta$		$2\xi-\phi-3\eta$		·4	·4	·4	·406	·152	− ·3
$2\tau+\xi-3\eta$		$2\xi+\phi-3\eta$			− ·1		− ·318	− ·257	+ ·1
$\tau-\xi-\eta$	187	$\xi-\phi-\eta$	171		− ·8		− ·135	− ·206	− ·1
$\tau-\xi+\eta$	188	$\xi-\phi+\eta$	172	− ·4	− ·8	− ·4	− ·135	− ·206	− ·1
$\tau+\xi-\eta$	189	$\xi+\phi-\eta$	173	− ·1	− ·4	− ·1	− ·585	− ·510	+ ·1
$\tau+\xi+\eta$	190	$\xi+\phi+\eta$	174	− ·7	− ·4	− ·7	− ·405	− ·403	·0
$\tau+\xi_{,}-\eta$	193	$\xi+\phi'-\eta$	175	·6	·6	·8	·951[5]	.648	− ·3
$\tau+\xi_{,}+\eta$	194	$\xi+\phi'+\eta$	176	·6	·6	·8	·951	·648	− ·3
$2\xi+\xi_{,}-\eta$	201	$2\phi+\phi'-\eta$					− ·257	− ·218	·0
$2\xi+\xi_{,}+\eta$	202	$2\phi+\phi'+\eta$					− ·445	− ·440	·0
$2\xi-\xi_{,}-\eta$	207	$2\phi-\phi'-\eta$					·257	·340	+ ·1
$2\xi-\xi_{,}+\eta$	208	$2\phi-\phi'+\eta$					·445	·440	·0
$4\tau-\eta$	241	$4\xi-\eta$	183	3·6	3·0	3·7	3·402[6]	3·553	+ ·2
$4\tau+\eta$	242	$4\xi+\eta$	184	1·2	·9	1·2	·366	·364	·0
$4\tau-\xi-\eta$	243	$4\xi-\phi-\eta$	185	6·3	6·4	6·3	6·548	6·561	·0
$4\tau-\xi+\eta$	244	$4\xi-\phi+\eta$	186	3·1	2·7	3·0	1·399	2·340	+ ·9
$4\tau-\xi_{,}-\eta$	246	$4\xi-\phi'-\eta$	188	·4	·2	·3	·197	·196	·0
$4\tau-2\xi-\eta$		$4\xi-2\phi-\eta$	190		·4	·2	·357	·354	·0
$4\tau-2\xi+\eta$		$4\xi-2\phi+\eta$	191	2·4	2·6[a]	2·4	1·237	1·226	·0
$4\tau-\xi-\xi_{,}-\eta$		$4\xi-\phi-\phi'-\eta$		·5	·4	·4	·315	·312	·0
$4\tau-\xi+\xi_{,}-\eta$		$4\xi-\phi+\phi'-\eta$		− ·2	− ·1	− ·2	− ·135	− ·134	·0
$4\tau-3\eta$		$4\xi-3\eta$		·1	·1	·1	·074	·117	·0

NOTES BY M. DE PONTÉCOULANT, VOL. IV. P. 631.

[4] Il y a erreur dans la réduction en nombres de la formule de M. Plana. T. I. p. 720, au lieu de 0″·313(
on doit lire 0″·626(4); ce qui, au lieu de 0″·627, donne 0″·940 pour le coefficient de sin $(2\xi-2\phi'-\eta)$, rés
qui concorde avec le nôtre.

[5] Dans la réduction en nombres des formules de M. Plana (t. I. p. 722), au lieu de −0″·033(5), on d
ire 0″·331(5) dans les coefficiens de sin $(\xi+\phi'-\eta)$ et de sin $(\xi+\phi'+\eta)$; ce qui donne, au lieu de 0″·951 po
ces coefficiens, 0″·653, résultat qui s'accorde avec le nôtre.

[6] M. Plana a omis, dans sa formule analytique, le terme principal de l'ordre $m^3\gamma$.

(*a*) Les résultats marqués d'un [a] dans ce tableau ont évidemment besoin de correction.

NOTES BY SIR J. LUBBOCK.

The Americans use Plana's expression for the latitude, in an altered form, with the exception of the coe
cient of sin η, and two small coefficients determined empirically by the Astronomer Royal.

According to the last determination of the Astronomer Royal, the coefficient of sin η is 18463″·1.

TABLE III.

Comparison of the Numerical Values of the Coefficients in the Expression for the Parallax of the Moon.

Lubbock. Argument.	Lubbock. No.	Pontécoulant. Argument.	Pontécoulant. No.	Bürg.	Burckhardt.	Damoiseau.	Plana.	Pontécoulant.	Adams.	Pontécoulant — Plana.
				″	″	″	″	″	″	″
	0		0	3421·0	3420·5	3420·89	3423·153	3421·153	3422·48	−2·0
2τ	1	2ξ		27·9	28·2	28·54	27·593	27·475	28·23	− ·1
ξ	2	ϕ		186·7	186·4	186·48	186·654	186·553	186·54	− ·1
$2\tau-\xi$	3	$2\xi-\phi$		34·4	34·5	36·43	33·921	34·175	34·30	+ ·2
$2\tau+\xi$	4	$2\xi+\phi$		3·0	3·1	3·05	3·075	3·060	3·09	·0
$\xi_,$	5	ϕ'	5	− 0·3	− 0·5	− 0·32	− 0·428	− 0·428	− 0·40	·0
$2\tau-\xi_,$	6	$2\xi-\phi'$		1·9	1·0	1·92	2·165	1·876	1·92	− ·3
$2\tau+\xi_,$	7	$2\xi+\phi'$	15	− 0·3	0·6	− 0·26	− 0·545	− 0·328	− 0·31	·2
2ξ	8	2ϕ		9·9	10·1	10·24	10·249	10·212	10·17	·0
$2\tau-2\xi$	9	$2\xi-2\phi$		0·4		− 0·41	0·167[a]	− 0·275	− 0·31	− ·1
$2\tau+2\xi$	10	$2\xi+2\phi$		0·2	0·2	0·14	0·270	0·269	0·28	·0
$\xi+\xi_,$	11	$\phi+\phi'$		− 0·9	− 0·9	− 0·92	− 0·919	− 0·913	− 0·95	·0
$2\tau-\xi-\xi_,$	12	$2\xi-\phi-\phi'$		1·7	0·3	1·45	1·449	1·449	1·45	·0
$2\tau+\xi+\xi_,$	13	$2\xi+\phi+\phi'$		− 0·1			− 0·001[a]	− 0·041		·0
$\xi-\xi_,$	14	$\phi-\phi'$		1·1	1·1	1·20	1·060	1·053	1·16	·0
$2\tau-\xi+\xi_,$	15	$2\xi-\phi+\phi'$		− 0·2	0·9	− ·24	− 0·380	− 0·375	− 0·23	·0
$2\tau+\xi-\xi_,$	16	$2\xi+\phi-\phi'$		0·2	0·1	·18	0·188	0·196	0·22	·0
3ξ	20	3ϕ		0·2	0·5	·63	0·634	0·633	0·63	·0
$2\xi+\xi_,$	23	$2\phi+\phi'$		− 0·1	− 0·1		− 0·080	− 0·091	− 0·10	·0
$2\xi-\xi_,$	26	$2\phi-\phi'$		0·1	0·1		0·085	0·102	0·12	·0
$\xi-2\eta$	65	$\phi-2\eta$		− 0·8	0·8	− ·70	1·194	− 0·700	− 0·71	− ·5
τ	101	ξ		− 1·0	− 1·0	− ·98	− 0·925	− 0·923	− 0·95	·0
$\tau-\xi_,$	104	$\xi-\phi'$				·14	0·152	0·149		·0
4τ	131	4ξ		0·3	0·1	·21	0·136	0·195	0·26	·0
$4\tau-\xi$	132	$4\xi-\phi$		0·6	0·6	0·57	0·498	0·497	0·60	·0
$4\tau-2\xi$	136	$4\xi+2\phi$		0·3	0·4	0·40	0·325	0·308	0·37	·0

Note by M. de Pontécoulant.

(*a*) Il y a erreur dans le formule analytique de M. Plana, ainsi que dans sa réduction en nombres.

Notes by Sir John Lubbock.

The Americans use Plana's expression for the Parallax.

Mr. Adams' values are given in vol. xiii. of the Memoirs of the Royal Astronomical Society, and are founded upon the correction of a numerical error in the Introduction to Burckhardt's Tables.

According to the last determination of the Astronomer Royal, the constant is $3423''\cdot54$.

TABLE IV.

Comparison of the Differences from Observation of Moon's Places, as indicated by Burckhardt's Tables and the American Tables.

Date of Obsn. with Greenwich Transit Circle.	R. A. B — O	R. A. A — O	Dec. B — O	Dec. A — O
1856.	s.	s.	"	"
Jan. 2	....	....	— 7·1	— 2·3
12	0·56	— 0·06	7·7	2·2
1 13	0·46	0·11	8·0	2·4
14	0·49	0·16	9·8	3·2
15	0·37	— 0·09	10·6	2·3
18	0·10	— 0·44	7·0	— 0·6
19	0·05	— 0·31	6·7	0·3
24	0·21	0·16	— 2·8	— 2·7
25	0·17	0·16	— 2·3	— 0·9
27	0·01	0·01	— 2·6	1·0
28	0·15	0·10	— 4·1	0·9
29	0·20	0·07	— 4·6	1·4
Feb. 14	0·56	0·03	6·5	— 0·7
2 15	0·46	— 0·14	3·1	— 2·9
16	0·33	— 0·13	2·8	— 1·4
24	0·20	0·11	— 5·2	0·1
25	0·09	0·03	— 3·8	1·9
26	0·13	0·09	— 4·1	2·2
Mar. 13	0·34	0·01	4·7	— 1·0
14	0·60	0·08	1·4	— 2·2
17	(0·35)	0·02	— 0·9	0·8
3 22	0·18	— 0·14	— 5·5	2·1
26	0·13	— 0·29	— 7·3	0·6
28	0·47	— 0·09	— 3·6	2·1
29	0·25	— 0·59	— 2·8	1·0
31	0·09	— 0·83	....	....
April 9	0·35	— 0·22	3·2	— 5·4
12	0·62	0·22	— 1·1	— 1·2
4 13	0·67	0·21	— 3·5	— 1·0
15	0·61	0·23	— 5·9	— 0·5
16	0·05	— 0·22	— 5·9	— 0·6
20	0·03	— 0·23	— 5·8	1·6
21	0·14	— 0·41	— 6·5	2·1
22	0·55	— 0·29	— 6·0	3·0
27	0·41	— 0·70	1·5	— 1·7
May 10	0·50	0·28	1·2	0·3
11	0·42	0·24	— 2·3	— 1·0
13	0·64	0·34	— 8·0	— 2·9
14	0·57	0·34	—10·0	— 4·8
17	0·06	0·25	— 6·1	— 2·5
18	— 0·05	0·08	— 2·9	1·7
5 19	0·03	— 0·06	— 3·6	2·0
20	0·42	0·00	— 3·3	3·2
22	0·50	— 0·58	1·1	4·1
26	0·88	— 0·40	7·8	— 3·1
ne 10	0·33	0·25	— 6·8	— 3·6
6 11	0·31	0·27	— 8·5	— 4·5
14	0·07	0·35	— 7·1	— 2·1
15	— 0·17	— 0·02	— 7·1	— 1·1
17	0·35	— 0·02	0·3	8·1
19	....	....	— 1·5	3·6
20	0·88	— 0·14	1·9	3·2
25	0·59	— 0·36	9·9	— 3·5
26	0·32	— 0·38	14·1	1·2

Date of Obsn. with Greenwich Transit Circle.	R. A. B — O	R. A. A — O	Dec. B — O	Dec. A — O
1856.	s.	s.	"	"
July 6	0·94	0·42	....	....
16	1·06	— 0·06	— 4·0	3·5
17	1·29	0·02	— 4·5	0·7
18	1·37	0·04	0·4	2·2
22	0·82	— 0·49	6·5	— 5·8
25	0·44	— 0·67	9·4	— 1·6
26	0·64	— 0·47	....	....
Aug. 5	0·70	0·29	— 2·7	1·8
6	0·31	0·12	— 0·5	3·0
7	0·19	0·20	— 3·9	— 0·5
10	0·38	0·24	— 5·6	0·0
12	0·73	0·08	— 4·5	2·3
7 13	0·89	— 0·05	— 3·4	2·1
14	1·13	— 0·17	— 1·1	1·6
15	1·35	— 0·15	2·0	0·9
22	0·82	— 0·43	9·2	0·3
Sept. 5	0·51	0·32	....	....
12	0·81	— 0·06	2·7	2·2
8 14	0·89	— 0·24	7·3	— 1·6
15	0·94	— 0·13	9·9	— 1·9
16	0·95	0·16	10·5	— 0·6
18	0·84	0·27	9·0	— 0·7
19	1·32	0·39	7·8	0·1
21	1·33	— 0·18	0·9	— 0·3
23	0·86	— 0·07	— 4·2	— 1·4
Oct. 12	0·09	— 0·35	5·5	1·3
13	0·36	— 0·22	7·8	— 0·1
16	1·10	0·69	10·1	2·6
18	....	....	3·9	— 0·1
21	1·30	0·23	— 5·0	0·7
Nov. 4	0·41	— 0·20	2·2	2·3
5	0·27	— 0·42	....	....
6	0·59	— 0·19	2·5	— 0·4
8	0·27	— 0·34	2·8	— 0·1
10	— 0·04	— 0·31	6·8	1·7
11	0·15	— 0·19	7·3	0·0
14	1·06	0·36	4·9	0·1
15	0·95	0·27	0·3	— 1·8
16	1·10	0·39	— 3·3	— 1·9
18	....	....	— 4·8	1·3
Dec. 2	0·65	0·02	2·6	— 3·5
7	0·43	— 0·35	8·3	1·3
9	0·26	— 0·43	8·4	0·8
10	....	....	4·6	— 1·5
13	0·92	0·25	1·7	4·0
14	0·81	0·03	— 0·4	0·7
9 15	0·62	— 0·08	— 0·4	2·1
16	0·73	0·16	— 4·0	0·6
17	0·64	0·23	— 5·5	— 0·3
18	0·58	0·26	— 6·3	— 0·6

Comparison of the Differences, etc.—*Continued.*

Date of Obsn. with Greenwich Transit Circle.	R. A.		Dec.	
	B − O	A − O	B − O	A − O
1857.	s.	s.	″	″
Jan. 2	0·51	− 0·28	3·4	− 3·3
3	0·64	− 0·15	5·3	− 3·3
8	....	....	2·4	− 3·3
11	0·74	0·03	− 1·0	− 0·2
13	0·63	− 0·04	− 1·0	3·7
10 { 14	0·63	0·07	− 2·4	2·6
15	0·30	− 0·09	− 2·8	1·9
16	0·44	0·16	− 6·2	− 2·0
31	0·50	− 0·39	8·6	− 1·7
Feb. 4	0·98	− 0·49	4·9	− 0·8
6	....	....	1·4	0·4
7	0·55	− 0·16	− 1·2	− 1·1
8	0·58	0·03	− 1·7	0·3
10	0·57	0·04	− 2·5	2·3
11	0·47	0·01	− 2·4	2·8
12	0·47	0·12	− 2·0	2·5
13	0·26	− 0·01	− 4·2	− 0·4
15	0·04	− 0·27	− 6·0	− 2·5
16	0·11	− 0·20	0·4	3·8
Mar. 1	....	....	10·3	0·5
4	1·16	− 0·08	....	....
5	1·05	− 0·11	0·6	0·1
6	0·64	− 0·30	− 1·6	− 0·1
7	0·45	− 0·26	− 1·8	0·0
8	0·39	− 0·14	− 4·2	− 0·7
9	....	....	− 6·5	− 2·1
12	0·44	0·17	− 5·2	0·2
15	0·17	− 0·16	− 2·6	1·0
11 { 16	0·11	− 0·24	2·5	5·8
17	0·24	− 0·16	− 0·1	2·5
31	0·47	− 0·29	2·1	− 1·6
April 2	0·59	− 0·21	− 0·4	0·4
3	0·75	− 0·03	− 3·0	− 0·2
6	0·13	− 0·30	− 6·2	− 1·1
8	0·11	− 0·15	− 3·3	1·6
9	0·36	0·12	− 4·4	0·5
14	0·35	− 0·27	2·0	3·6
18	0·18	− 0·69	− 2·1	− 4·8
May 1	0·49	− 0·03	− 2·3	0·4
3	0·62	0·10	− 6·8	− 1·3
4	0·41	− 0·05	− 8·1	− 1·9
5	0·27	− 0·04	− 5·4	0·4
6	0·24	0·07	− 5·5	− 0·5
7	0·09	− 0·01	− 4·6	0·0
8	− 0·01	− 0·17	− 1·9	2·8
12 { 11	0·41	− 0·36	2·6	5·3
13	0·77	− 0·16	4·1	2·4
14	0·67	− 0·24	6·2	2·4
16	0·78	− 0·14	....	....
17	0·38	− 0·51	8·0	− 1·1
27	0·72	− 0·12	4·7	3·2
31	0·58	0·17	− 5·9	− 0·9
June 1	0·35	− 0·01	− 9·1	− 3·3
2	0·56	0·26	− 7·5	− 1·5
4	0·16	0·09	− 3·9	1·2
5	− 0·12	− 0·10	− 1·3	3·6
8	0·16	− 0·23	− 0·3	3·6
9	0·60	− 0·05	2·7	4·5
10	0·73	− 0·18	4·0	3·0
13 { 12	0·83	− 0·36	8·2	− 0·3
13	1·13	− 0·11	12·7	1·0
14	1·01	− 0·20	15·0	1·2
15	0·67	− 0·41	14·1	− 0·2
1857.	s.	s.	″	″
June 25	1·08	− 0·02	− 1·5	2·0
26	0·94	0·03	− 3·0	1·3
27	0·58	− 0·04	− 2·6	1·9
28	0·36	− 0·02	− 2·8	1·1
29	0·44	0·23	− 3·2	0·2
July 6	0·33	0·03	2·4	6·9
7	0·53	0·12	1·8	5·1
14 { 9	1·00	0·18	4·0	2·5
11	1·09	− 0·28	8·8	− 1·8
12	1·09	− 0·48	10·4	− 4·5
14	....	....	14·1	− 1·2
31	0·24	0·16	0·5	1·8
Aug. 2	0·45	0·13	0·0	2·2
3	0·31	− 0·21	1·6	3·8
4	0·61	− 0·04	4·5	6·4
9	0·82	− 0·63	7·0	− 6·0
25	0·00	− 0·16	− 2·2	− 0·1
26	− 0·04	0·02	....	....
29	− 0·04	0·09	4·0	3·6
15 { 30	0·12	0·18	3·4	4·2
31	0·42	0·31	4·3	5·5
Sept. 1	0·45	0·10	3·7	4·6
4	0·98	0·21	6·7	1·4
5	0·99	− 0·17	8·3	− 0·1
6	0·88	− 0·24	8·3	− 3·3
11	1·48	− 0·50	− 2·6	− 1·8
24	0·26	0·29	....	....
28	0·00	0·11	4·7	5·8
16 { 29	0·18	0·23	3·4	5·4
30	0·03	0·01	3·2	5·8
Oct. 1	0·06	− 0·06	4·3	4·5
2	0·12	− 0·29	4·3	1·3
6	1·24	− 0·28	9·7	1·1
8	1·41	− 0·35	0·8	− 0·9
9	1·53	− 0·47	− 3·9	− 1·1
23	0·44	0·10	....	....
25	0·23	− 0·05	7·1	4·9
26	0·25	− 0·13	6·4	4·3
17 { 27	0·26	− 0·05	5·9	3·8
29	0·01	− 0·06	7·9	7·6
30	0·06	− 0·03	5·3	4·1
31	0·25	− 0·19	7·2	3·3
Nov. 1	0·54	− 0·33	8·2	0·0
5	1·26	0·24	− 1·9	− 2·2
22	0·58	− 0·14	6·5	3·4
24	0·57	− 0·18	5·2	− 0·9
27	0·85	0·20	8·4	1·3
18 { 28	0·70	0·18	9·2	2·4
29	0·56	0·01	9·7	2·6
30	0·70	− 0·19	7·5	0·2
Dec. 1	0·94	− 0·40	4·9	− 0·9
2	0·98	− 0·43	2·0	− 0·6
3	0·92	− 0·22	− 1·9	− 1·3
4	0·73	− 0·02	− 3·9	− 1·5
7	0·55	0·01	− 9·0	− 1·6
10	0·29	− 0·03	− 9·4	− 2·3
25	0·90	− 0·23	7·0	− 3·9
27	1·04	0·11	7·2	− 0·9
28	1·07	0·25	5·6	− 0·3
19 { 29	(1·40)	0·51	7·2	3·5
30	1·06	− 0·05	0·2	− 1·1
31	0·79	− 0·47	− 1·8	0·2

Comparison of the Differences, etc.—*Continued.*

Date of Obsn. with Greenwich Transit Circle.	R. A. B—O	R. A. A—O	Dec. B—O	Dec. A—O
1858.	s.	s.	"	"
Jan. 1	0·95	— 0·24	— 8·2	— 2·5
6	0·50	0·15	— 5·9	— 1·2
9	0·13	— 0·29	— 1·1	1·7
19	0·52	— 0·15	2·3	— 3·2
20	....	....	— 0·7	— 6·4
22	0·38	— 0·49	5·1	— 2·6
23	0·53	— 0·63	4·2	— 3·6
24	0·78	— 0·56	3·2	— 3·1
20 { 25	0·95	— 0·35	1·0	— 2·5
26	0·98	— 0·18	— 0·4	— 1·2
27	1·18	0·08	— 2·2	— 0·1
28	1·15	0·02	— 4·1	0·7
31	0·91	— 0·18	— 5·3	6·4
Feb. 1	0·84	— 0·13	— 6·1	4·5
3	0·42	— 0·30	— 3·8	1·7
4	0·68	0·01	— 2·3	1·1
6	0·41	— 0·19	4·9	4·2
7	0·33	— 0·05	— 1·3	— 3·3
17	0·40	— 0·32	....	....
18	0·28	— 0·54	4·4	— 3·7
19	0·35	— 0·55	4·4	— 2·8
20	0·52	— 0·50	7·6	2·0
21 { 21	0·74	— 0·39	3·4	— 0·2
22	0·75	— 0·54	1·6	0·6
24	1·17	— 0·25	— 4·5	0·8
25	1·23	— 0·10	— 8·5	— 0·4
27	0·77	— 0·22	— 8·2	3·1
Mar. 6	0·50	— 0·27	2·3	1·2
8	0·42	0·11	8·0	5·4
21	0·60	— 0·38	0·8	— 0·3
22	0·98	0·07	— 4·6	— 4·1
22 { 23	1·33	0·30	— 1·6	1·3
24	1·32	0·17	— 6·9	— 1·1
25	1·33	0·14	—11·0	— 3·5
26	0·81	— 0·32	— 9·6	0·0
27	....	....	— 8·4	2·0
29	0·39	— 0·10	— 5·0	2·7
April 1	0·43	— 0·06	0·8	2·8
3	0·38	— 0·20	1·6	0·1
18	0·57	— 0·24	— 1·3	— 1·2
19	0·64	0·00	0·1	1·9
20	0·74	0·13	— 2·5	0·8
21	0·79	0·11	— 4·7	0·9
22	0·80	0·08	— 9·5	— 2·1
23	0·71	— 0·02	— 9·5	— 0·9
24	0·61	— 0·02	— 8·3	0·7
25	0·24	— 0·27	— 6·9	1·3
26	0·25	— 0·10	— 8·1	— 1·5
28	0·27	0·03	— 3·6	— 0·7
May 5	0·46	0·25	6·7	2·5
6	0·20	0·15	7·7	4·3
7	— 0·11	0·00	6·7	4·6
16	0·96	— 0·24	....	....
18	0·41	— 0·18	— 0·1	3·6
19	0·35	— 0·11	— 3·1	1·6
22	0·34	0·00	— 5·2	2·0
23	— 0·09	— 0·41	— 7·5	— 0·2
25	0·29	0·03	— 3·2	2·1
27	....	....	— 1·0	1·9
30	0·25	— 0·15	3·8	1·8

Date of Obsn. with Greenwich Transit Circle.	R. A. B—O	R. A. A—O	Dec. B—O	Dec. A—O
1858.	s.	s.	"	"
June 1	0·41	— 0·01	15·5	10·1
3	0·45	0·19	10·8	3·9
7	— 0·28	— 0·38	....	....
15	0·90	— 0·26	— 1·5	5·2
16	0·52	— 0·24	....	....
17	0·19	— 0·29	— 2·0	3·6
18	0·35	0·05	— 2·3	2·7
21	0·44	0·20	— 3·2	0·7
22	0·50	0·25	0·7	4·0
24	— 0·13	— 0·31	2·1	4·1
23 { 25	0·05	— 0·17	5·2	6·9
27	0·30	— 0·09	7·1	6·7
28	0·72	0·24	2·8	0·6
30	....	....	7·0	0·3
July 1	0·86	0·27	9·1	0·8
14	....	....	— 8·2	3·7
15	0·72	— 0·10	— 6·4	2·9
18	0·34	0·11	0·9	2·4
19	0·54	0·35	1·6	1·7
21	0·40	0·26	5·1	4·0
25	0·20	0·03	5·1	4·1
26	0·41	0·11	4·3	2·1
24 { 28	0·70	0·17	4·2	— 1·7
30	0·61	— 0·33	8·0	— 2·3
31	0·93	— 0·27	10·0	— 1·2
Aug. 1	1·13	— 0·36	10·3	— 0·8
2	1·31	— 0·40	9·4	— 0·2
4	0·95	— 0·58	9·5	7·1
13	0·99	0·18	— 9·6	1·6
19	0·44	0·62	6·3	3·3
25 { 20	0·13	0·28	9·8	7·6
22	0·02	0·04	6·5	5·0
23	0·06	— 0·04	4·9	2·4
24	0·41	0·19	5·4	1·9
25	0·51	0·16	7·0	1·5
27	0·60	— 0·23	6·1	— 3·3
28	0·78	— 0·40	6·4	— 4·3
30	0·96	— 0·96	7·4	— 0·2
31	1·35	— 0·66	5·7	2·3
Sept. 1	1·25	— 0·62	— 2·9	— 2·8
12	0·91	0·13	— 4·4	0·8
13	0·58	0·30	— 1·7	— 1·3
14	0·35	0·50	....	....
15	0·05	0·43	2·1	— 0·4
16	— 0·33	0·15	5·8	3·8
26 { 18	— 0·18	0·27	5·7	5·1
21	— 0·10	— 0·26	8·2	4·6
22	0·34	0·01	7·5	2·6
23	0·47	0·03	7·8	1·5
24	0·67	0·18	7·8	0·8
25	0·85	0·13	6·9	— 0·5
30	1·34	— 0·35	— 7·7	— 2·1
Oct. 2	1·09	— 0·04	—11·0	— 1·0
15	— 0·23	0·14	5·4	3·8
27 { 16	— 0·28	0·14	8·0	7·7
17	— 0·19	0·37	5·1	6·2
20	— 0·06	— 0·10	7·9	4·7
22	0·92	0·07	7·8	— 0·5
28	1·22	— 0·28	— 7·3	0·9
29	1·22	— 0·23	—14·0	— 2·9

COMPARISON OF THE DIFFERENCES, ETC.—*Continued.*

Date of Obsn. with Greenwich Transit Circle.	R. A.		Dec.		Date of Obsn. with Greenwich Transit Circle.	R. A.		Dec.	
	B—O	A—O	B—O	A—O		B—O	A—O	B—O	A—O
1858.	s.	s.	″	″	1858.	s.	s.	″	″
Nov. 11	0·52	0·47	8·5	5·4	Dec. 15	....	....	12·1	4·4
28 { 12	0·18	0·24	6·3	3·0	17	0·96	0·13	8·8	0·5
13	— 0·06	0·04	7·4	4·3	18	1·15	0·04	7·7	0·3
14	....	....	5·6	2·7	19	1·49	— 0·16	5·2	0·2
15	— 0·01	0·18	8·6	5·8	20	....	....	3·5	3·8
17	— 0·09	— 0·08	8·7	4·2	21	1·40	— 0·76	— 5·1	1·1
18	0·45	0·13	9·4	2·8	22	....	....	— 7·9	1·6
19	....	....	3·6	— 5·6	29 { 23	0·88	0·00	—10·1	— 0·9
20	0·93	— 0·55	8·5	— 0·7	24	0·65	0·22	— 9·1	— 1·4
21	1·17	— 0·80	4·0	— 2·4	26	0·72	0·24	—13·1	— 4·3
22	1·27	— 0·73	1·3	0·4	27	0·65	— 0·10	—11·8	— 2·6
23	1·17	— 0·46	— 5·7	— 2·2	28	0·75	— 0·08	— 9·3	— 1·3
26	0·87	0·10	—13·4	— 3·7					

The Americans employ the following equations in the longitude, which are given by the Astronomer Royal in vol. xvii. of the *Memoirs of the Royal Astronomical Society:*

$+ 6''\cdot 38$ sin. longitude of moon's node,

$—\ \ ''\cdot 97$ cos. longitude of moon's node.

The former coefficient M. de Pontécoulant made $6''\cdot 623$; the latter term is not recognised by theory. Recently, the Astronomer Royal has found these coefficients to be $6''\cdot 44$ and $—1\cdot 06$; the former being almost identical with M. de Pontécoulant's value. They also employ two other small corrections depending upon the node, taken from the Astronomer Royal.

Of 90 coefficients employed by the Americans in their expression for the longitude, 57 are identical with those of M. de Pontécoulant; the remainder of the equations employed by the Americans, with only two exceptions, differ very slightly from those of M. de Pontécoulant.

The Americans also employ the following equations in the latitude, taken from the Astronomer Royal, p. 56, *u* being the moon's true longitude: $2''\cdot 17 \cos u \quad —8''\cdot 80 \sin u$.

The first of these terms is not required by theory; the coefficient of the latter, according to M. de Pontécoulant, is $—7''\cdot 867$. Recently, the Astronomer Royal has found these coefficients to be $1''\cdot 93$ and $—8''\ 59$.

Of 63 coefficients employed by the Americans in their expression for the latitude, 29 are identical with those of M. de Pontécoulant; the remainder, with four exceptions, differ very slightly from those of M. de Pontécoulant. In those four cases, the coefficients they employ are certainly erroneous.

The following are the differences which exist at present between M. de Pontécoulant's coefficients in the expression for the longitude

and those employed in the American Tables arranged in their order of magnitude:

	P.—A.	The Americans have taken the coefficients from
Arg. 27.	$-2''\cdot6 \sin(2\tau-2\xi+\xi_{,})$	Longstreth.
„ 5.	$1\cdot4 \sin \xi_{,}$	Airy, 1848.
	$-\cdot9 \sin(4\tau-2\xi-\xi_{,})$	Longstreth.
„ 9.	$-\cdot6 \sin(2\tau-2\xi)$	Plana.
„ 62.	$-\cdot6 \sin 2\eta$	Plana.
„ 2.	$\cdot5 \sin \xi$	Airy, 1848.
„ 6.	$-\cdot5 \sin(2\tau-\xi_{,})$	Plana.
„ 14.	$-\cdot4 \sin(\xi-\xi_{,})$	Plana.
„ 15.	$\cdot3 \sin(2\tau-\xi+\xi_{,})$	Plana.
„ 39.	$-\cdot3 \sin(2\tau-4\xi)$	Plana.
„ 77.	$-\cdot3 \sin(2\xi-2\eta)$	Plana.
„ 101.	$-\cdot3 \sin \tau$	Plana.
„ 132.	$.3 \sin(4\tau-\xi)$	Plana.

Neglecting all, of which the coefficient is less than $''\cdot3$.

The above, therefore, is the quantity which should be added to the longitude of the American tables, in order to obtain the longitude that would be given by the coefficients of M. de Pontécoulant, when quantities under $\cdot3''$ are neglected.

If we adopt the Astronomer Royal's recent determination of the parallactic inequality, viz., $124''\cdot37$, we must add to the longitude of the American tables,

$$-2''\cdot3 \sin \tau.$$

These corrections are so small, that practically the places given by the American tables may be considered as identical with those which would be given by tables founded upon Pontécoulant's coefficients, and therefore as founded upon our theory, with the exception of Hansen's two erroneous equations due to the action of Venus. The period of these last is so long, that they may be considered as merely affecting the longitude of the epoch; and the Astronomer Royal's three empirical corrections not embraced by theory are so small, as not sensibly to affect the result.

The Americans profess to have employed the Astronomer Royal's value of the inclination given in the "Reduction of the Greenwich Observations," which is $18535''.46$; the Astronomer Royal's recent determination of this quantity, viz., $18535''\cdot55$, is so close to the

former, that the difference may be neglected.

The Americans employ the epochs for which astronomers are indebted to the Astronomer Royal, and which were given by him in vol. xvii. of the *Memoirs of the Royal Astronomical Society*. Mr. Main has kindly computed for me the following table, at the request of my friend the Astronomer Royal. The quantities are for Washington, mean noon, Jan. 0 of the years 1801, 1821, 1841, 1861, and 1881.

Epoch of Longitude.		Epoch of Mean Anomaly.		Arg. of Latitude.	
Damoiseau corrected by Airy.	American Tables.	Damoiseau corrected by Airy.	American Tables.	Damoiseau corrected by Airy.	American Tables.
° ′ ″	″	° ′ ″	″	° ′ ″	″
107 55 40·3	40·5	201 50 49·2	49·2	93 59 45·2	47·9
241 30 20·5	20·7	241 36 41·9	41·9	254 24 4·4	7·0
15 5 0·6	0·9	281 22 34·6	34·6	54 48 23·4	26·1
148 39 40·8	41·1	321 8 27·6	27·6	215 12 42·7	45·2
282 14 20·9	21·3	0 54 20·0	20·0	15 37 1·6	4·3

The Astronomer Royal has kindly furnished me with the following epoch of mean longitude, which I think it desirable to place on record.

Correction for 1806......................	=	0″·00
Correction for 50 years $=50\times+''\!\cdot596$	=	29″·80
Correction for 1856..	=	29″·80
(in centesimals)	=	91``·97

		°		``
Damoiseau's epoch for 1856...	=	221	18	06·8
Corrected epoch for 1856......	=	221	18	98·8
		°	′	″
(in sexagesimals)	=	199	4	15·2

The following Table is intended to convert the differences contained in Table IV. of Right Ascension and Declination into differences of Longitude and Latitude.

The Astronomer Royal has given a very extended Table for converting errors of Right Ascension and North Polar distance into errors of Longitude and Ecliptic Polar distance in the appendix to vol. of the Greenwich observations for 1836. This Table extends over nearly fifty 4to. pages, and is carried to ·001. The Table in the next page, calculated by Mr. Farley, will be found sufficient for converting small differences of R. A. and Dec. into the corresponding differences of Longitude and Latitude for bodies moving in or near

the ecliptic when only tenths of seconds of space are retained. In the Table of the Astronomer Royal, the R. A. is supposed to be given in time, the numbers in the two first columns of Table V. are the same as his P. and R. divided by 1·5.

The numbers in the first column are the only ones which vary sensibly for 5° of Latitude, and the small correction to be applied is as follows :—

R. A.	Correction of first col. of Table V. for		R. A.	R. A.	Correction of first col. of Table V. for		R. A.
	5° Lat. N.	5° Lat. S.			5° Lat. N.	5° Lat. S.	
°	"	"	°	°	"	"	°
0	0	0	0	180	0	0	180
30	— ·2	+ ·2	30	210	+ ·2	— ·2	210
60	— ·3	+ ·3	60	240	+ ·3	— ·3	240
90	— ·3	+ ·3	90	270	+ ·3	— ·3	270
120	— ·3	+ ·3	120	300	+ ·3	— ·3	300
150	— ·2	+ ·2	150	330	+ ·2	— ·2	330
180	0	0	180	360	0	·0	360

The Table reads thus; required the change of Longitude corresponding to $+13''$ of R. A., the Moon's R. A. being 60°, and Latitude 5° N.

Table V. gives $+9''\!\cdot\!2$ for $10''$ of R. A.
From above — ·3

+8·9

The change of Longitude for $+13''$ of R. A. is therefore

$$+\ 8{\cdot}9\times\frac{13}{10}=+11''{\cdot}57$$

TABLE V.

Change of Moon's Longitude and Latitude, caused by a change of +10″ in Right Ascen. and Declin.

(Assuming her to move in the Ecliptic).

R. A.	+10″ in Right Ascen.		+10″ in Declination.		R. A.
	Long.	Lat.	Long.	Lat.	
°	″	″	″	″	°
0	+ 9·2	— 4·0	+ 4·0	+ 9·2	0
10	+ 9·2	— 3·9	+ 4·0	+ 9·2	10
20	+ 9.2	— 3·7	+ 3·8	+ 9·3	20
30	+ 9·2	— 3·4	+ 3·5	+ 9·4	30
40	+ 9.2	— 3·0	+ 3·1	+ 9·5	40
50	+ 9.2	— 2·5	+ 2·6	+ 9·7	50
60	+ 9·2	— 1·9	+ 2·0	+ 9·8	60
70	+ 9·2	— 1·3	+ 1·4	+ 9·9	70
80	+ 9.2	— 0·7	+ 0·7	+10·0	80
90	+ 9·2	0·0	0·0	+10·0	90
100	+ 9·2	+ 0·7	— 0·7	+10·0	100
110	+ 9·2	+ 1·3	— 1·4	+ 9·9	110
120	+ 9·2	+ 1·9	— 2·0	+ 9·8	120
130	+ 9·2	+ 2·5	— 2·6	+ 9·7	130
140	+ 9·2	+ 3·0	— 3·1	+ 9·5	140
150	+ 9·2	+ 3·4	— 3·5	+ 9·4	150
160	+ 9·2	+ 3·7	— 3·8	+ 9·3	160
170	+ 9·2	+ 3·9	— 4·0	+ 9·2	170
180	+ 9·2	+ 4·0	— 4·0	+ 9·2	180
190	+ 9·2	+ 3·9	— 4·0	+ 9·2	190
200	+ 9·2	+ 3·7	— 3·8	+ 9·3	200
210	+ 9·2	+ 3·4	— 3·5	+ 9·4	210
220	+ 9·2	+ 3·0	— 3·1	+ 9·5	220
230	+ 9·2	+ 2·5	— 2·6	+ 9·7	230
240	+ 9·2	+ 1·9	— 2·0	+ 9·8	240
250	+ 9·2	+ 1·3	— 1·4	+ 9·9	250
260	+ 9·2	+ 0·7	— 0·7	+10·0	260
270	+ 9·2	0·0	0·0	+10·0	270
280	+ 9·2	— 0·7	+ 0·7	+10·0	280
290	+ 9·2	— 1·3	+ 1·4	+ 9·9	290
300	+ 9·2	— 1·9	+ 2·0	+ 9·8	300
310	+ 9·2	— 2·5	+ 2·6	+ 9·7	310
320	+ 9·2	— 3·0	+ 3·1	+ 9·5	320
330	+ 9·2	— 3·4	+ 3·5	+ 9·4	330
340	+ 9·2	— 3·7	+ 3·8	+ 9·3	340
350	+ 9·2	— 3·9	+ 4·0	+ 9·2	350
360	+ 9·2	— 4·0	+ 4·0	+ 9·2	360

Example of the Use of the above Table:

Suppose the Moon in about Right Ascension 313½°, required the change of Longitude and Latitude corresponding to +15″·65 of Right Ascension, and —5″·05 of Declination.

For Longitude.

Opposite 313½° in col. 2 is +9″·2 for +10″ of R. A., hence for +15″·65, +14″·40
″ ″ 4 ″ +2 ·37 ″ ″ Dec., ″ — 5 ·05 — 1 ·40

+13 ·0

For Latitude.

Opposite 313½° in col. 3 is —2″·67 for +10″ of R. A., hence for +15″·65, — 4″·18
″ ″ 5 ″ +9 ·63 ″ ″ Dec., ″ — 5 ·05 — 4 ·86

— 9 ·0

+13″ and —9″ agrees with the converse operation in following page.

TABLE VI.

Change of Moon's Right Ascension and Declination, caused by a change of +10″ in Long. and Lat.

(Assuming her to move in the Ecliptic).

Long.	+10″ in Long.		+10″ in Lat.		Long.
	R. A.	Dec.	R. A.	Dec.	
°	″	″	″	″	°
0	+ 9·2	+ 4·0	− 4·0	+ 9·1	0
10	+ 9·2	+ 3·9	− 4·0	+ 9·2	10
20	+ 9·4	+ 3·8	− 3·9	+ 9·3	20
30	+ 9·6	+ 3·5	− 3·6	+ 9·4	30
40	+ 9·8	+ 3·2	− 3·3	+ 9·5	40
50	+10·1	+ 2·7	− 2·8	+ 9·6	50
60	+10·4	+ 2·1	− 2·3	+ 9·7	60
70	+10·7	+ 1·5	− 1·6	+ 9·8	70
80	+10·9	+ 0·8	− 0·8	+ 9·9	80
90	+10·9	0·0	0·0	+10·0	90
100	+10·9	− 0·8	+ 0·8	+ 9·9	100
110	+10·7	− 1·5	+ 1·6	+ 9·8	110
120	+10·4	− 2·1	+ 2·3	+ 9·7	120
130	+10·1	− 2·7	+ 2·8	+ 9·6	130
140	+ 9·8	− 3·2	+ 3·3	+ 9·5	140
150	+ 9·6	− 3·5	+ 3·6	+ 9·4	150
160	+ 9·4	− 3·8	+ 3·9	+ 9·3	160
170	+ 9·2	− 3·9	+ 4·0	+ 9·2	170
180	+ 9·2	− 4·0	+ 4·0	+ 9·1	180
190	+ 9·2	− 3·9	+ 4·0	+ 9·2	190
200	+ 9·4	− 3·8	+ 3·9	+ 9·3	200
210	+ 9·6	− 3·5	+ 3·6	+ 9·4	210
220	+ 9·8	− 3·2	+ 3·3	+ 9·5	220
230	+10·1	− 2·7	+ 2·8	+ 9·6	230
240	+10·4	− 2·1	+ 2·3	+ 9·7	240
250	+10·7	− 1·5	+ 1·6	+ 9·8	250
260	+10·9	− 0·8	+ 0·8	+ 9·9	260
270	+10·9	0·0	0·0	+10·0	270
280	+10·9	+ 0·8	− 0·8	+ 9·9	280
290	+10·7	+ 1·5	− 1·6	+ 9·8	290
300	+10·4	+ 2·1	− 2·3	+ 9·7	300
310	+10·1	+ 2·7	− 2·8	+ 9·6	310
320	+ 9·8	+ 3·2	− 3·3	+ 9·5	320
330	+ 9·6	+ 3·5	− 3·6	+ 9·4	330
340	+ 9·4	+ 3·8	− 3·9	+ 9·3	340
350	+ 9·2	+ 3·9	− 4·0	+ 9·2	350
360	+ 9·2	+ 4·0	− 4·0	+ 9·1	360

Example of the Use of the above Table:

Suppose the Moon in about Longitude 311°, required the change of Right Ascension and Declination corresponding to +13″ of Longitude and −9″ of Latitude.

For Right Ascension.

Opposite 311° in col. 2 is +10″·07 for +10″ of long., hence for +13,″ +13″·09
″ ″ ″ 4 ″ − 2 ·85 ″ ″ lat., ″ ″ − 9, + 2 ·56

+15 ·65

For Declination.

Opposite 311° in col. 3 is +2″·75 for +10″ of long., hence for +13″, + 3″·58
″ ″ ″ 5 ″ +9 ·59 ″ ″ lat., ″ ″ − 9, − 8 ·63

− 5 ·05

For the converse operation, see preceding page.

The Americans use the following values of the constants derived, for the mean noon of Washington of the date 1801, Jan. 0, from the values obtained by AIRY in his Memoir upon the *Corrections of the Elements of the Moon's Orbit*, published in the *Memoirs of the Royal Astronomical Society*, vol. xvii., and from Bessel's Tables of the sun:

		Daily motion.	
	° ′ ″	° ′ ″	°
Moon's mean long. .	107 55 40·5	13 10 35·0281	or 13·176397
Long. of moon's perigee	*266 4 51·3	6 41·0578	·111405
Sun's mean long. . .	279 52 44·3	59 8·3300	·985647
Long. of sun's perigee .	279 31 10·4	·16947	·000047
Long. of moon's node .	13 55 52·6	− 3 10·63366	−·052954

Washington being 5h. 8m. 12s. W. of Greenwich, the time from Washington noon of Jan. 0 to Greenwich noon of Jan. 1, is 18h. 51m. 48s., or ·785972: this gives for Greenwich mean noon, Jan. 1, 1801, and throwing out the even circumferences,

			Motion	
	° ′ ″	°		°
Moon's mean long. .	118 17 3·1	or 118·2841	in 20453 days is	216·8478
Long. of moon's perigee	266 10 6·5	266·1684	"	118·5664
Sun's mean long. . .	280 39 13·2	280·6603	"	359·4381
Long. of sun's perigee .	279 31 10·5	279·3050	"	·9612
Long. of moon's node .	13 53 22·8	13·8896	"	3·0682

Hence for Greenwich, 1857, Jan. 0,

	°	°
Moon's mean long. .	335·1319	r = 55·0335
Long. of moon's perigee.	24·7348	ξ = 310·3971
Sun's mean long. . .	280·0984	$\xi_{,}$ = 359·6176
Long. of sun's perigee .	280·4808	η = 324·3105
Long. of moon's node .	10·8214	

With these quantities, Mr. Farley constructed the following tables.

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* Misprinted in the American Tables, 166°.

TABLE VII.

Longitude Arguments, etc.

Lubbock. No.	Lubbock. Argument.	Epoch, M. Noon at Gh. Jan. 0, 1857.	Age.	Period.	Daily Motion.
		°	d.	d.	°
1	2τ	110·06	4·51	14·7653	24·381500
2	ξ	310·39	23·76	27·5546	13·064992
3	$2\tau-\xi$	159·67	14·11	31·8119	11·316508
4	$2\tau+\xi$	60·46	1·61	9·6137	37·446492
5	ξ_1	359·62	364·87	365·2598	0·985600
6	$2\tau-\xi_1$	110·45	4·72	15·3873	23·395900
7	$2\tau+\xi_1$	109·68	4·32	14·1916	25·367100
8	2ξ	260·79	9·98	13·7773	26·129984
9	$2\tau-2\xi$	209·27	86·20	205·8927	— 1·748484
10	$2\tau+2\xi$	10·86	0·22	7·1271	50·511484
11	$\xi+\xi_1$	310·01	22·06	25·6217	14·050592
12	$2\tau-\xi-\xi_1$	160·05	15·49	34·8469	10·330908
13	$2\tau+\xi+\xi_1$	60·08	1·56	9·3672	38·432092
14	$\xi-\xi_1$	310·78	25·73	29·8028	12·079392
15	$2\tau-\xi+\xi_1$	159·29	12·95	29·2633	12·302108
16	$2\tau+\xi-\xi_1$	60·85	1·67	9·8736	36·460892
17	$2\xi_1$	359·24	182·24	182·6299	1·971200
18	$2\tau-2\xi_1$	110·83	4·95	16·0640	22·410300
19	$2\tau+2\xi_1$	109·30	4·15	13·6608	26·352700
20	3ξ	211·19	5·39	9·1848	39·194976
21	$2\tau-3\xi$	258·88	6·83	24·3022	— 14·813476
22	$2\tau+3\xi$	321·26	5·05	5·6625	63·576476
23	$2\xi+\xi_1$	260·41	9·60	13·2765	27·115584
24	$2\tau-2\xi-\xi_1$	209·66	54·99	131·6712	— 2·734084
26	$2\xi-\xi_1$	261·18	10·39	14·3173	25·144384
27	$2\tau-2\xi+\xi_1$	208·89	198·06	471·8934	— 0·762884
28	$2\tau+2\xi-\xi_1$	11·24	0·23	7·2689	49·525884
29	$\xi+2\xi_1$	309·63	20·59	23·9422	15·036192
30	$2\tau-\xi-2\xi_1$	160·43	17·17	38·5220	9·345308
32	$\xi-2\xi_1$	311·16	28·05	32·4506	11·093792
33	$2\tau-\xi+2\xi_1$	158·91	11·96	27·0927	13·287708
34	$2\tau+\xi-2\xi_1$	61·23	1·73	10·1479	35·475292
38	4ξ	161·59	3·09	6·8886	52·259968
39	$2\tau-4\xi$	308·48	1·85	12·9132	— 27·878468
41	$3\xi+\xi_1$	210·81	5·25	8·9596	40·180576
42	$2\tau-3\xi-\xi_1$	259·26	6·38	22·7861	— 15·799076
44	$3\xi-\xi_1$	211·57	5·54	9·4218	38·209376
45	$2\tau-3\xi+\xi_1$	258·49	7·34	26·0344	— 13·827876
48	$2\tau-2\xi-2\xi_1$	210·04	40·32	96·7824	— 3·719684
62	2η	288·62	10·91	13·6061	26·458702
63	$2\tau-2\eta$	181·45	85·96	173·3101	— 2·077202
64	$2\tau+2\eta$	38·69	0·76	7·0810	50·840202
65	$\xi-2\eta$	21·78	25·25	26·8783	— 13·393710
66	$\xi+2\eta$	239·02	6·05	9·1085	39·523694
67	$2\tau-\xi-2\eta$	231·05	8·52	23·7746	— 15·142194
68	$2\tau-\xi+2\eta$	88·29	2·34	9·5301	37·775210
69	$2\tau+\xi-2\eta$	131·84	12·00	32·7636	10·987790
70	$2\tau+\xi+2\eta$	349·09	5·46	5·6333	63·905194

LONGITUDE ARGUMENTS, ECT.—*Continued.*

Lubbock. No.	Lubbock. Argument.	Epoch, M. Noon at Gh. Jan. 0, 1857.	Age.	Period.	Daily Motion.
		°	d.	d.	°
71	$\xi,-2\eta$	71·00	11·34	14·1325	— 25·473102
72	$\xi,+2\eta$	288·24	10·50	13·1175	27·444302
73	$2\tau-\xi,-2\eta$	181·83	58·17	117·5394	— 3·062802
74	$2\tau-\xi,+2\eta$	39·07	0·78	7·2210	49·854602
75	$2\tau+\xi,-2\eta$	181·06	163·92	329·7912	— 1·091602
77	$2\xi-2\eta$	332·17	84·66	1095·1638	— 0·328718
78	$2\xi+2\eta$	189·42	3·61	6·8455	52·588686
79	$2\tau-2\xi-2\eta$	280·65	2·81	12·7627	— 28·207186
80	$2\tau-2\xi+2\eta$	137·89	5·58	14·5689	24·710218
81	$2\tau+2\xi-2\eta$	82·24	3·42	14·9671	24·052782
83	$\xi+\xi,-2\eta$	21·39	27·29	29·0133	— 12·408110
84	$\xi+\xi,+2\eta$	238·64	5·89	8·8869	40·509294
89	$\xi-\xi,-2\eta$	22·16	23·49	25·0350	— 14·379310
90	$\xi-\xi,+2\eta$	239·40	6·21	9·3414	38·538094
101	τ	55·03	4·51	29·5306	12·190750
102	$\tau-\xi$	104·63	292·10	411·7853	— 0·874242
103	$\tau+\xi$	5·43	0·22	14·2542	25·255742
104	$\tau-\xi,$	55·42	4·95	32·1281	11·205150
105	$\tau+\xi,$	54·65	4·15	27·3217	13·176350
106	$\tau-2\xi$	154·24	14·76	25·8264	— 13·939234
107	$\tau+2\xi$	315·83	8·24	9·3944	38·320734
108	$\tau-\xi-\xi,$	105·02	137·10	193·5648	— 1·859842
109	$\tau+\xi+\xi,$	5·05	0·19	13·7188	26·241342
110	$\tau-\xi+\xi,$	104·25	936·23	3233·0260	0·111358
116	3τ	165·10	4·51	9·8435	36·572250
117	$3\tau-\xi$	214·70	9·13	15·3144	23·507258
118	$3\tau+\xi$	115·50	2·33	7·2526	49·637242
121	$3\tau-2\xi$	264·31	25·31	34·4752	10·442266
125	$3\tau-\xi+\xi,$	214·32	8·75	14·6982	24·492858
131	4τ	220·13	4·51	7·3826	48·763000
132	$4\tau-\xi$	269·74	7·56	10·0846	35·698008
133	$4\tau+\xi$	170·53	2·76	5·8226	61·827992
134	$4\tau-\xi,$	220·52	4·62	7·5349	47·777400
135	$4\tau+\xi,$	219·75	4·42	7·2364	49·748600
136	$4\tau-2\xi$	319·34	14·11	15·9059	22·633016
137	$4\tau+2\xi$	120·93	1·61	4·8069	74·892984
138	$4\tau-\xi-\xi,$	270·12	7·78	10·3709	34·712408
140	$4\tau-\xi+\xi,$	269·35	7·34	9·8136	36·683608
	$\xi-4\eta$	93·16	6·70	9·0333	— 39·852412
	$4\tau-3\xi$	8·94	0·93	37·6253	9·568024
	$4\tau-2\xi-\xi,$	319·72	14·77	16·6302	21·647416
	$4\tau-2\xi+\xi,$	318·96	13·50	15·2422	23·618616
	$4\tau-2\eta$	291·51	13·07	16·1404	22·304298
	$4\tau-\xi-2\eta$	341·12	36·92	38·9640	9·239306
	$4\tau+\xi-2\eta$	241·91	6·84	10·1783	35·369290
	$6\tau-\xi$	19·80	0·33	5·9921	60·079508
	$6\tau-2\xi$	69·41	1·48	7·6572	47·014516

TABLE VIII.

Latitude Arguments, etc.

Lubbock. No.	Lubbock. Argument.	Epoch, M. Noon at Gh. Jan. 0, 1867.	Age.	Period.	Daily Motion.
		°	d.	d.	°
146	η	324·31	24·51	27·2122	13·229351
147	$2\tau-\eta$	145·76	13·07	32·2808	11·152149
148	$2\tau+\eta$	74·38	1·98	9·5717	37·610851
149	$\xi-\eta$	346·09	84·63	2190·3271	− 0·164359
150	$\xi+\eta$	274·71	10·45	13·6912	26·294343
151	$2\tau-\xi-\eta$	195·36	86·07	188·2015	− 1·912843
152	$2\tau-\xi+\eta$	123·98	5·05	14·6664	24·545859
153	$2\tau+\xi-\eta$	96·15	3·97	14·8655	24·217141
154	$2\tau+\xi+\eta$	24·77	0·49	7·1040	50·675843
155	$\xi,-\eta$	35·31	26·52	29·4028	− 12·243751
156	$\xi,+\eta$	323·93	22·79	25·3254	14·214951
157	$2\tau-\xi,-\eta$	146·14	14·37	35·4103	10·166549
158	$2\tau-\xi,+\eta$	74·76	2·04	9·8293	36·625251
159	$2\tau+\xi,-\eta$	145·37	11·98	29·6595	12·137749
160	$2\tau+\xi,+\eta$	73·99	1·92	9·3273	38·596451
161	$2\xi-\eta$	296·48	22·98	27·9056	12·900633
162	$2\xi+\eta$	225·10	5·72	9·1465	39·359335
163	$2\tau-2\xi-\eta$	244·96	7·68	24·0355	− 14·977835
164	$2\tau-2\xi+\eta$	173·58	15·12	31·3565	11·480867
165	$2\tau+2\xi-\eta$	46·55	1·25	9·6561	37·282133
166	$2\tau+2\xi+\eta$	335·17	5·26	5·6479	63·740835
167	$\xi+\xi,-\eta$	345·70	420·95	438·3610	0·821241
168	$\xi+\xi,+\eta$	272·32	10·06	13·1965	27·279943
169	$2\tau-\xi-\xi,-\eta$	195·74	56·67	124·2046	− 2·898443
170	$2\tau-\xi-\xi,+\eta$	124·36	5·28	15·2799	23·560259
171	$2\tau+\xi+\xi,-\eta$	95·77	3·80	14·2842	25·202741
173	$\xi-\xi,-\eta$	346·47	11·77	313·0545	− 1·149959
174	$\xi-\xi,+\eta$	275·09	10·87	14·2243	25·308743
175	$2\tau-\xi+\xi,-\eta$	194·98	177·97	388·2477	− 0·927243
176	$2\tau-\xi+\xi,+\eta$	123·60	4·84	14·1003	25·531459
177	$2\tau+\xi-\xi,-\eta$	96·52	4·16	15·4962	23·231541
178	$2\tau+\xi-\xi,+\eta$	25·16	0·51	7·2449	49·690243
179	$2\xi,-\eta$	34·93	28·87	31·9768	− 11·258151
180	$2\xi,+\eta$	323·55	21·29	23·6834	15·200551
181	$2\tau-2\xi,-\eta$	146·52	15·96	39·2116	9·180949
182	$2\tau-2\xi,+\eta$	75·14	2·11	10·1011	35·639651
183	$2\tau+2\xi,-\eta$	144·99	11·05	27·4320	13·123349
185	$\tau-\eta$	90·72	259·27	346·6202	− 1·038601
186	$\tau+\eta$	19·34	0·76	14·1620	25·420101
195	$3\xi-\eta$	246·88	9·51	13·8645	25·965625
196	$3\xi+\eta$	175·50	3·35	6·8670	52·424327
	3η	252·93	6·37	9·0707	39·688053
	$\xi-3\eta$	57·46	11.36	13·5221	− 26·623061
	$2\tau-3\xi-\eta$	294·57	2·33	12·8375	− 28·042827
	$2\tau-3\xi+\eta$	223·19	86·36	227·2548	− 1·584125
	$2\tau-3\eta$	217·14	9·33	23·5194	− 15·306553
	$2\tau+3\eta$	3·00	0·05	5·6189	64·069553
	$2\tau-\xi-3\eta$	266·74	3·29	12·6887	− 28·371545

LATITUDE ARGUMENTS, ETC.—*Continued.*

Lubbock. No.	Lubbock. Argument.	Epoch, M. Noon at Gh. Jan. 0, 1857.	Age.	Period.	Daily Motion.
		°	d.	d.	°
	$2\tau+\xi-3\eta$	167·53	85·86	160·6024	— 2·241561
187	$\tau-\xi-\eta$	140·33	15·57	25·5254	— 14·103593
188	$\tau-\xi+\eta$	68·95	5·58	29·1377	12·355109
189	$\tau+\xi-\eta$	41·12	3·42	29·9342	12·026391
190	$\tau+\xi+\eta$	329·74	8·57	9·3543	38·485093
193	$\tau+\xi_{,}-\eta$	90·34	508·78	6792·3230	— 0·053001
194	$\tau+\xi_{,}+\eta$	18·96	0·72	13·6334	26·405701
201	$2\xi+\xi_{,}-\eta$	296·10	21·32	25·9249	13·886233
202	$2\xi+\xi_{,}+\eta$	224·72	5·57	8·9230	40·344935
207	$2\xi-\xi_{,}-\eta$	296·87	24·92	30·2139	11·915033
208	$2\xi-\xi_{,}+\eta$	225·49	5·88	9·3814	38·373735
241	$4\tau-\eta$	255·82	7·20	10·1312	35·533649
242	$4\tau+\eta$	184·44	2·98	5·8072	61·992351
243	$4\tau-\xi-\eta$	305·43	13·59	16·0223	22·468657
244	$4\tau-\xi+\eta$	234·05	4·78	7·3578	48·927359
246	$4\tau-\xi_{,}-\eta$	256·21	7·42	10·4203	34·548049
	$4\tau-2\xi-\eta$	355·03	37·35	38·2830	9·403665
	$4\tau-2\xi+\eta$	283·65	7·91	10·0384	35·862367
	$4\tau-\xi-\xi_{,}-\eta$	305·81	14·23	16·7574	21·483057
	$4\tau-\xi+\xi_{,}-\eta$	305·04	13·01	15·3490	23·454257
	$4\tau-3\eta$	327·20	36·06	39·6697	9·074947

TABLE IX.

		LONGITUDE.								LATITUDE.	
No. of Group.	Date.	a	b	A—O l	$a+l$	$b+l$	l^2	$(a+l)^2$	$(b+l)^2$	A—O l'	l'^2
	1856. d	″	″	″	″	″				″	
1	Jan. 13.98	0.4	− 0.6	1.4	1.8	0.8	1.96	3.24	0.64	2.1	4.41
2	Feb. 15.34	1.4	− 0.7	− 1.2	0.2	− 1.9	1.44	0.04	3.61	− 1.7	2.89
3	Mar. 23.24	2.0	− 0.6	− 2.6	− 0.6	− 3.2	6.76	0.36	10.24	− 0.2	0.04
4	April 13.31	2.0	− 0.5	1.0	3.0	0.5	1.00	9.00	0.25	− 1.4	1.96
5	May 19.71	1.3	− 0.2	− 1.2	0.1	− 1.4	1.44	0.01	1.96	1.5	2.25
6	June 12.83	0.7	0.0	4.1	4.8	4.1	16.81	23.04	16.81	− 1.4	1.96
7	Aug. 13.93	− 1.3	0.5	− 0.7	− 2.0	− 0.2	0.49	4.00	0.04	1.9	3.61
8	Sept. 14.77	− 1.9	0.6	− 1.2	− 3.1	− 0.6	1.44	9.61	0.36	− 0.1	0.01
9	Dec. 16.17	− 0.6	0.4	1.4	0.8	1.8	1.96	0.64	3.24	1.8	3.24
	1857.										
10	Jan. 14.46	0.4	0.2	0.0	0.4	0.2	0.00	0.16	0.04	1.3	1.69
11	Mar. 16.70	1.9	− 0.3	− 3.3	− 1.4	− 3.6	10.89	1.96	12.96	2.7	7.29
12	May 11.18	1.6	− 0.6	− 3.1	− 1.5	− 3.7	9.61	2.25	13.69	2.4	5.76
13	June 14.28	0.6	− 0.7	− 1.7	− 1.1	− 2.4	2.89	1.21	5.76	3.0	9.00
14	July 9.60	− 0.3	− 0.6	− 0.8	− 1.1	− 1.4	0.64	1.21	1.96	2.0	4.00
15	Aug. 30.35	− 1.8	− 0.3	3.1	1.3	2.8	9.61	1.69	7.84	4.1	16.81
16	Sept. 29.88	− 2.0	0.0	2.8	0.8	2.8	7.84	0.64	7.84	4.7	22.09
17	Oct. 28.34	− 1.8	0.2	0.4	− 1.4	0.6	0.16	1.96	0.36	4.9	24.01
18	Nov. 28.91	− 1.0	0.4	1.3	0.3	1.7	1.69	0.09	2.89	1.4	1.96
19	Dec. 29.46	− 0.1	0.6	1.0	0.9	1.6	1.00	0.81	2.56	0.3	0.09
	1858.										
20	Jan. 25.36	0.9	0.7	− 4.4	− 3.5	− 3.7	19.36	12.25	13.69	− 1.4	1.96
21	Feb. 21.58	1.6	0.6	− 5.7	− 4.1	− 5.1	32.49	16.81	26.01	0.1	0.01
22	Mar. 23.83	2.0	0.5	2.7	4.7	3.2	7.29	22.09	10.24	− 1.2	1.44
23	June 25.69	0.3	− 0.3	− 0.2	0.1	− 0.5	0.04	0.01	0.25	4.5	20.25
24	July 28.60	− 0.9	− 0.5	− 0.7	− 1.6	− 1.2	0.49	2.56	1.44	0.6	0.36
25	Aug. 21.42	− 1.6	− 0.6	4.0	2.4	3.4	16.00	5.76	11.56	3.9	15.21
26	Sept. 18.77	− 2.0	− 0.7	2.3	0.3	1.6	5.29	0.09	2.56	2.5	6.25
27	Oct. 16.30	− 2.0	− 0.6	4.7	2.7	4.1	22.09	7.29	16.81	4.5	20.25
28	Nov. 14.54	− 1.4	− 0.5	2.3	0.9	1.8	5.29	0.81	3.24	3.5	12.25
29	Dec. 25.72	− 0.3	− 0.2	2.2	1.9	2.0	4.84	3.61	4.00	− 1.6	2.56
	29)	− 1.9	− 3.8	7.9	6.0	4.1	190.81	133.20	182.85	44.7	193.61
		− 0.1	− 0.1	0.3	0.2	0.1	6.58	4.59	6.30	1.5	6.68

$a = 2''.0 \sin \xi_,$ $\qquad b = -''.7 \sin(2r - 2\xi + \xi_,)$

TABLE X.

COMPARISON OF THE DIFFERENCES FROM OBSERVATION OF MOON'S PLACES, AS INDICATED BY HANSEN'S TABLES AND THE AMERICAN TABLES.

Date of Obsn. with Greenwich Transit Circle	Long.		Lat.	
	H−O	A−O	H−O	A−O
1856.	″	″	″	″
Jan. 12	− 3·5	0·1	0·6	2·4
13	− 1·8	2·5	0·0	1·7
14	0·0	3·5	0·5	2·2
15	− 2·1	− 0·6	1·1	2·6
18	− 2·6	− 6·2	0·3	− 0·1
19	− 1·6	− 4·2	1·6	0·2
24	0·1	3·3	− 2·2	1·6
25	− 0·1	2·7	− 1·6	0·1
27	− 2·0	− 0·2	− 2·1	1·0
28	− 0·6	1·1	− 1·7	1·4
29	− 1·0	0·6	− 1·1	0·5
Feb. 14	− 1·1	0·5	0·3	0·6
15	− 1·2	− 1·9	− 0·8	− 2·9
16	0·3	− 1·7	1·2	− 1·6
24	− 0·8	1·6	− 2·5	0·7
25	− 2·8	− 0·2	− 1·4	2·1
26	− 2·2	0·7	− 0·8	2·5
Mar. 13	− 5·0	0·1	0·8	− 1·0
14	− 2·3	1·3	0·2	− 2·1
22	− 0·8	− 2·7	− 0·4	1·2
26	− 5·9	− 4·2	− 2·7	− 0·1
28	− 0·8	− 1·3	0·5	2·2
29	− 1·7	− 8·0	0·6	2·1
April 9	− 3·0	− 3·5	− 2·8	− 5·1
12	− 2·0	3·3	1·0	− 0·5
13	− 2·7	3·4	1·5	0·0
15	− 1·8	3·4	2·8	0·8
16	− 6·7	− 2·8	0·0	− 1·9
20	− 2·9	− 3·9	− 1·5	0·6
21	− 3·8	− 6·2	− 2·2	0·5
22	− 0·9	− 4·6	− 1·3	2·1
27	− 3·9	−10·1	1·0	1·3
May 10	− 0·7	3·8	2·0	1·4
11	− 3·7	3·6	0·7	0·2
13	− 3·2	5·9	0·7	− 0·7
14	− 2·7	6·6	− 0·7	− 2·4
17	− 1·7	4·2	− 0·6	− 1·1
18	− 3·4	0·6	1·2	2·0
19	− 2·9	− 1·3	− 0·4	1·8
20	− 0·2	− 0·4	− 0·2	3·3
22	− 3·4	− 7·7	1·0	4·8
26	0·4	− 6·7	− 1·2	− 0·6
June 10	− 3·3	4·9	− 1·4	− 1·8
11	− 3·0	5·6	− 1·7	− 3·5
14	− 2·0	5·4	0·5	− 0·4
15	− 3·7	0·0	− 1·0	− 1·1
17	− 2·3	− 0·9	5·6	8·1
20	− 0·1	− 1·2	0·3	3·5
25	2·2	− 6·3	− 1·2	− 1·2
26	4·3	− 4·8	2·6	3·1
1856.	″	″	″	″
July 16	0·3	− 0·5	1·5	3·6
17	0·7	0·4	− 2·0	0·6
18	1·7	1·2	− 0·3	2·0
22	2·1	− 9·1	− 1·4	− 2·4
25	2·2	− 9·7	0·6	0·8
Aug. 5	− 1·6	3·3	0·8	3·4
6	− 4·2	0·5	0·8	3·5
7	− 3·0	2·9	− 2·3	0·6
10	− 4·3	3·3	− 0·7	0·5
12	− 4·6	1·2	0·4	2·3
13	− 4·0	− 0·2	0·2	2·2
14	− 1·8	− 2·0	0·0	2·2
15	0·7	− 1·7	− 0·1	1·5
22	5·1	− 5·8	1·4	1·4
Sept. 12	− 1·8	0·1	− 1·0	2·4
14	0·5	− 4·0	− 0·4	0·0
15	2·3	− 2·6	0·1	− 1·0
16	4·4	2·0	0·2	− 1·4
18	3·1	3·6	− 0·1	− 1·5
19	7·4	5·5	− 0·3	− 0·6
21	5·3	− 2·5	− 1·1	− 0·6
23	3·2	− 0·5	− 1·6	− 1·6
Oct. 12	− 4·8	− 4·3	0·5	− 0·9
13	− 1·2	− 3·1	− 0·4	1·1
16	3·7	10·0	1·2	− 1·3
21	7·8	2·9	1·2	1·7
Nov. 4	− 4·5	− 2·3	0·4	2·9
6	− 2·4	− 2·7	− 1·3	0·6
8	− 4·7	− 4·8	− 1·7	2·0
10	− 4·3	− 3·7	1·5	3·3
11	− 3·3	− 2·6	0·2	0·8
14	2·1	5·0	− 0·1	0·1
15	1·3	3·9	− 2·0	− 1·4
16	4·9	6·0	− 2·3	− 0·7
Dec. 2	− 2·7	− 0·7	1·6	− 3·6
7	− 2·8	− 4·3	1·8	3·1
9	− 4·2	− 5·7	1·6	2·4
13	1·3	2·8	− 0·6	4·5
14	0·9	0·3	− 1·1	0·8
15	− 1·0	− 1·7	0·2	1·6
16	2·6	2·0	− 0·9	1·4
17	2·6	3·3	− 1·2	1·1
18	2·7	3·9	− 0·6	1·0

Comparison of the Differences, etc.—*Continued.*

Date of Obsn. with Greenwich Transit Circle.	Long.		Lat.	
	H—O	A—O	H—O	A—O
1857.	″	″	″	″
Jan. 2	− 3·8	− 5·2	− 0·2	− 1·4
3	− 1·7	− 3·4	− 0·8	− 2·2
11	0·0	0·5	− 1·2	0·0
13	− 1·8	− 2·0	0·5	3·2
14	0·0	− 0·1	0·2	2·7
15	− 3·1	− 2·0	− 1·0	1·2
16	1·2	3·0	− 2·6	− 0·9
31	− 4·4	− 6·1	0·9	0·6
Feb. 4	− 3·2	− 6·8	1·1	− 0·5
7	− 0·6	− 1·9	− 0·1	− 1·7
8	0·4	0·4	0·2	0·4
10	− 1·0	− 0·4	0·5	2·3
11	− 1·8	− 0·9	0·6	2·6
12	− 0·2	− 0·3	0·9	3·0
13	− 0·7	0·0	− 2·2	− 0·4
15	− 3·7	− 2·9	− 4·4	− 3·7
16	− 5·0	− 3·6	2·5	3·2
Mar. 5	− 1·7	− 1·6	2·1	− 0·1
6	− 4·0	− 4·1	1·7	− 1·1
7	− 3·2	− 3·6	2·1	− 1·2
8	− 1·4	− 1·7	0·5	− 1·4
12	0·2	2·3	− 0·5	1·1
15	− 2·7	− 2·5	− 0·9	0·4
16	− 5·0	− 4·2	3·9	0·0
17	− 3·1	− 2·4	1·6	2·1
31	− 4·1	− 4·0	− 1·1	− 1·6
April 2	− 3·9	− 3·0	1·2	− 0·3
3	− 1·7	− 0·4	1·7	− 0·3
6	− 5·2	− 3·7	− 0·9	− 2·8
8	− 5·1	− 2·8	0·5	0·6
9	− 1·9	1·5	0·1	1·0
14	− 4·2	− 3·8	1·5	3·5
18	− 1·5	− 11·1	− 2·4	7·8
May 1	− 1·2	− 0·6	1·0	0·2
3	− 1·2	1·9	1·0	− 0·6
4	− 3·0	0·1	− 0·6	− 2·0
5	− 3·9	− 0·7	0·7	0·1
6	− 2·8	1·1	− 0·3	− 0·1
7	− 3·9	− 0·2	− 1·4	− 0·1
8	− 6·5	− 3·1	− 0·3	− 0·4
11	− 5·1	− 5·2	1·1	5·1
13	− 0·5	− 1·9	− 0·3	2·8
14	− 1·3	− 2·7	0·9	3·2
17	0·2	− 7·5	1·1	1·9
27	0·2	− 2·4	2·1	2·9
31	− 1·2	2·8	0·6	0·2
June 1	− 3·6	1·1	− 2·5	− 3·1
2	− 0·8	4·2	0·6	0·1
4	− 3·3	0·9	0·1	1·6
5	− 5·8	− 2·3	0·5	3·2
8	− 4·8	− 3·2	− 0·9	3·7
9	− 1·0	− 0·4	0·2	4·6
10	− 1·0	− 1·8	− 0·1	3·4
12	− 0·9	− 5·1	0·2	1·6
13	4·4	− 1·2	0·8	1·6
14	4·8	− 2·3	1·7	2·3
15	3·0	− 5·8	1·8	2·3
25	2·7	− 0·9	− 0·3	1·8
1857.	″	″	″	″
June 26	− 1·0	0·0	− 0·3	1·4
27	− 3·7	− 1·3	− 0·4	1·5
28	− 5·0	− 0·7	− 0·4	0·9
29	− 2·7	3·2	0·7	1·6
July 6	− 2·2	1·0	3·0	6·9
7	− 1·4	2·6	1·0	4·7
9	2·6	3·3	− 0·8	1·6
11	4·1	− 4·6	− 0·4	0·0
12	3·6	− 8·4	− 0·5	− 1·2
31	− 4·9	2·0	− 0·3	2·1
Aug. 2	− 3·2	1·8	− 0·8	2·1
3	− 6·0	− 2·4	− 1·1	4·2
4	− 3·2	1·0	− 2·9	6·1
9	3·5	− 11·0	− 0·5	− 1·9
25	− 6·0	− 2·2	− 3·5	− 0·8
29	− 7·5	1·3	1·2	3·6
30	− 6·3	2·9	1·1	3·9
31	− 3·1	5·3	1·5	4·5
Sept. 1	− 3·9	2·7	0·5	4·0
4	3·7	3·5	− 0·6	0·0
5	5·4	− 2·4	0·2	0·9
6	4·4	− 4·5	0·4	− 1·8
11	3·9	− 6·8	− 3·0	− 2·3
28	− 5·9	2·9	1·1	5·3
29	− 3·7	4·9	− 0·7	4·1
30	− 5·3	2·2	0·0	5·4
Oct. 1	− 4·6	1·0	0·8	4·5
2	− 4·5	− 3·5	0·3	2·0
6	3·6	− 3·6	2·9	1·9
8	3·1	− 4·8	− 0·7	− 1·0
9	4·2	− 6·2	− 1·9	− 1·9
25	− 4·6	0·3	1·9	4·9
26	− 4·6	− 0·6	0·9	4·7
27	− 4·7	0·6	− 0·1	3·8
29	− 4·3	2·2	2·2	7·4
30	− 3·4	1·2	0·0	4·0
31	− 1·4	− 1·3	1·1	4·1
Nov. 1	− 0·1	− 4·6	1·3	1·6
5	3·8	3·5	− 0·5	− 1·9
22	− 5·8	− 1·1	2·1	3·8
24	− 5·5	− 2·8	− 0·9	0·1
27	− 1·5	3·3	− 1·9	0·0
28	− 2·4	3·2	0·3	1·4
29	− 3·9	1·0	2·7	2·4
30	− 3·0	− 2·5	2·2	0·7
Dec. 1	− 0·9	− 5·6	1·4	− 0·3
2	0·4	− 5·9	0·1	− 0·7
3	1·8	− 2·9	− 0·9	− 1·7
4	1·8	0·1	− 0·9	− 1·5
7	2·6	0·8	− 0·2	− 1·4
10	0·1	0·4	− 1·3	− 2·3
25	− 4·8	− 4·8	− 0·3	− 2·3
27	− 5·1	1·3	0·9	− 1·3
28	− 4·4	3·4	1·6	− 0·9
29	1·0	6·7	5·3	3·2
30	− 2·8	− 0·7	0·7	− 1·2
31	− 5·6	− 6·6	− 0·6	− 1·2

COMPARISON OF THE DIFFERENCES, ETC.—*Continued.*

Date of Obsn. with Greenwich Transit Circle.	Long.		Lat.	
	H—O	A—O	H—O	A—O
1858.	"	"	"	"
Jan. 1	+ 0·1	− 2·6	− 4·5	− 3·4
6	+ 0·6	2·5	− 1·1	− 0·3
9	− 5·3	− 4·5	− 0·7	0·6
19	− 4·1	− 3·1	0·7	− 2·1
22	− 6·0	− 7·7	2·4	0·2
23	− 7·9	− 9·8	1·2	− 0·8
24	− 7·4	− 8·3	0·5	− 1·5
25	− 6·2	− 5·1	0·1	− 2·0
26	− 6·3	− 2·5	0·8	− 1·3
27	− 3·2	1·1	1·5	0·2
28	− 3·1	0·1	1·2	0·7
31	− 2·4	− 5·0	1·2	4·8
Feb. 1	− 1·0	− 3·6	− 0·4	3·3
3	− 4·7	− 4·8	− 1·9	− 0·1
4	− 1·6	− 0·2	− 0·4	1·1
6	− 6·0	− 3·4	3·1	3·6
7	− 4·3	− 0·4	− 3·9	− 3·4
18	− 5·5	− 8·8	1·0	− 0·5
19	− 5·3	− 8·5	0·6	− 0·2
20	− 4·6	− 6·4	3·5	3·6
21	− 6·0	− 5·4	0·9	0·5
22	− 9·0	− 7·4	1·4	0·8
24	− 4·7	− 3·5	1·6	0·1
25	− 1·5	− 1·2	0·0	− 0·8
27	− 2·1	− 4·2	0·2	1·6
Mar. 6	− 4·4	− 4·0	0·5	0·7
8	− 2·5	1·7	5·0	5·3
21	− 5·7	− 5·3	− 1·3	0·0
22	− 2·9	1·2	− 4·4	− 4·0
23	− 1·9	3·9	2·4	2·0
24	− 2·9	2·7	0·9	− 0·4
25	0·0	3·1	− 0·6	− 2·7
26	− 3·8	− 4·4	− 0·1	− 1·7
29	− 2·3	− 2·5	0·3	1·9
April 1	− 2·5	− 1·6	0·3	2·6
3	− 2·5	− 2·8	− 1·5	− 0·2
18	− 3·3	− 3·3	− 0·9	− 1·3
19	− 1·7	− 0·3	1·4	1·9
20	− 1·4	1·6	1·4	1·2
21	− 2·6	1·2	1·7	1·3
22	− 1·8	1·8	− 0·5	− 1·5
23	− 1·7	0·1	− 0·1	− 0·9
24	− 1·0	− 0·6	0·6	0·5
25	− 4·3	− 4·3	− 1·4	− 0·4
26	− 2·7	− 0·8	− 3·9	− 2·0
28	− 4·3	0·6	− 4·0	− 0·6
May 5	0·1	4·2	− 1·1	1·3
6	− 1·1	3·6	0·7	3·3
7	− 1·6	1·7	0·5	4·3
18	− 3·6	− 3·4	2·2	2·7
19	− 2·4	− 2·0	0·5	1·0
22	− 1·9	− 0·8	0·9	1·8
23	− 6·6	− 5·6	− 4·2	− 2·5
25	− 3·0	− 0·2	− 1·8	2·1
30	− 4·2	− 1·8	− 1·7	2·0

Date of Obsn. with Greenwich Transit Circle.	Long.		Lat.	
	H—O	A—O	H—O	A—O
1858.	"	"	"	"
June 1	1·1	2·6	7·9	9·7
3	0·7	4·0	0·7	2·6
15	− 5·1	− 5·2	1·8	3·7
17	− 6·0	− 5·6	0·1	1·6
18	− 2·6	− 0·3	0·8	2·8
21	− 1·2	2·5	− 1·7	1·6
22	− 0·9	2·4	0·6	4·8
24	− 6·9	− 4·6	− 1·5	3·6
25	− 5·7	− 2·4	1·5	6·9
27	− 1·6	0·0	2·8	6·9
28	2·3	3·5	− 3·1	− 0·3
July 1	3·9	4·1	− 1·2	− 0·8
15	− 3·9	− 2·5	− 1·0	2·1
18	− 5·7	0·7	0·0	2·8
19	− 3·2	4·3	0·2	3·0
21	− 3·0	3·2	0·6	4·4
25	− 0·7	1·4	0·6	3·9
26	0·7	2·1	− 0·7	1·6
28	2·6	1·8	− 2·3	− 2·5
30	0·6	− 5·5	0·9	− 0·1
31	2·9	− 4·3	0·8	0·4
Aug. 1	2·6	− 5·2	1·0	1·1
2	2·7	− 5·6	0·7	1·4
4	1·3	− 7·4	7·9	7·8
13	− 1·0	1·8	− 0·6	2·5
19	− 0·6	8·4	0·6	3·1
20	− 3·3	4·8	3·7	7·1
22	− 3·4	2·0	1·4	4·6
23	− 3·8	0·3	0·6	2·4
24	0·6	3·2	− 0·2	0·8
25	2·7	2·8	1·0	0·4
27	4·0	− 4·6	0·1	− 1·7
28	4·8	− 7·0	− 0·5	− 1·9
30	0·0	−12·9	2·2	2·7
31	2·1	− 8·4	1·8	2·4
Sept. 1	0·9	− 8·6	− 3·2	− 2·8
12	− 1·3	1·6	− 0·1	1·3
13	− 1·6	4·4	− 1·2	− 0·4
15	− 3·5	6·0	− 1·1	0·3
16	− 7·8	2·3	2·3	3·6
18	− 5·4	5·1	1·6	3·9
21	− 5·3	− 1·7	3·8	6·8
22	− 0·7	1·2	1·3	2·3
23	0·1	1·1	1·6	1·2
24	2·5	2·8	1·2	− 0·2
25	4·0	1·7	0·6	− 1·1
30	4·1	− 4·4	− 3·8	− 2·9
Oct. 2	4·3	− 0·2	− 1·5	− 1·1
15	− 5·2	2·8	1·5	3·3
16	− 4·4	4·1	3·6	6·8
17	− 3·7	7·1	0·0	4·2
20	− 4·6	0·5	1·6	4·9
22	1·9	0·8	− 1·1	− 0·8
28	3·3	− 4·1	0·4	0·0
29	5·1	− 2·3	− 2·8	− 3·8

COMPARISON OF THE DIFFERENCES, ETC.—*Continued.*

Date of Obsn. with Greenwich Transit Circle.	Long.		Lat.		Date of Obsn. with Greenwich Transit Circle.	Long.		Lat.	
	H—O	A—O	H—O	A—O		H—O	A—O	H—O	A—O
1858.	″	″	″	″	1858.	″	″	″	″
Nov. 11	— 2·0	7·6	2·9	4·1	Dec. 17	— 5·6	2·0	1·0	— 0·1
12	— 4·2	4·1	0·9	2·1	18	— 4·0	0·6	2·1	0·2
13	— 6·0	1·9	1·9	4·0	19	— 0·8	— 2·1	2·4	0·4
15	— 4·8	4·8	0·9	4·4	21	1·0	—10·7	— 0·8	— 0·6
17	— 7·9	0·6	1·0	4·3	23	2·9	0·3	— 1·8	— 0·9
18	— 2·3	2·8	— 0·4	0·9	24	1·4	3·6	— 0·2	— 0·1
20	— 2·0	— 7·8	1·4	1·2	26	2·7	1·6	— 2·1	— 2·5
21	— 1·1	—11·4	— 0·3	— 0·8	27	0·1	— 0·4	— 2·4	— 3·0
22	0·0	—10·1	0·9	0·6	28	1·1	— 0·7	— 1·6	— 1·6
23	0·6	— 6·1	— 2·8	— 2·9					
26	2·4	2·6	— 0·8	— 3·0					

The above Table shows the following results:—

Year.	No. of Obsns.	Mean Error with reference to Sign.				Mean Error without reference to Sign.			
		Long.		Lat.		Long.		Lat.	
		H—O	A—O	H—O	A—O	H—O	A—O	H—O	A—O
		″	″	″	″	″	″	″	″
1856	93	— 1·0	— 0·4	— 0·1	0·7	2·6	3·2	1·1	1·7
1857	115	— 1·8	— 1·3	0·2	1·2	3·1	2·8	1·1	2·2
1858	130	— 2·0	— 1·1	0·3	1·1	3·1	3·6	1·5	2·2
The 3 Years.	338	— 1·6	— 0·9	0·2	1·0	3·0	3·2	1·3	2·0

Prof. Hansen says that his "coefficients are, throughout, accurate to two hundredths of a second: many, he considers, to be still more accurate."* I think Prof. Hansen greatly mistaken in this estimate. In the preface, p. xi., which was written before I had seen the comparison here given, I have expressed my belief that there "is now no coefficient in the longitude of which the value is doubtful to the amount of ″1." My opinion now is very different, seeing how much Prof. Hansen's places and the American places differ from observation and from each other I think that many of the coefficients must be uncertain to the extent of ″1.

* *Monthly Notices*, November 10, 1854.